

APPLICATION OF MONTE CARLO SIMULATION IN THE FIELD OF MECHANICAL ENGINEERING

^aDANIEL BUC, ^bGABRIELA MASÁROVÁ

University of Žilina, Faculty of Operation and Economics Of Transport and Communication, Department of Economics, Univerzitná 1, 010 26 Žilina, Slovakia

email: ^adaniel.buc@fpedas.uniza.sk

email: ^bgabriela.masarova@fpedas.uniza.sk

The article is an output of scientific project VEGA 1/0357/11 Klieštík, T. and col.: Research on the possibility of applying fuzzy-stochastic approach and Corporate Metrics as tools of quantification and diversification of business risk.

Abstract: The paper deals with one of the often used computer simulation, mathematical-statistical method of individual variants and risk quantification - Monte Carlo. It is divided into two main parts. The first one is focused on the essence and characteristic features of this simulation, such as the sensitivity analysis, steps of simulation performance and its basic principles. The second part represents a Monte Carlo simulation example and its practical application in the field of industry engineering. According to mathematical formulas or statistical methods, there are Monte Carlo characteristics described, calculated and simulated in this part. The conclusion includes interpretation of success potential of a particular project.

Keywords: risk, Monte Carlo, variables, costs, standard deviation..

1 Features of Monte Carlo Simulation

The fundamental characteristic of simulations is the information usage from the complete distribution of the input variables. The indicators of project effectiveness are calculated according to this distribution. In this case, the values of input variables are derived from the defined distribution.

Monte Carlo method is used to quantify the likely and deterministic tasks based on multiple repeated random experiments. We construct probabilistic task with identical solution to the original task. The final solution has a probabilistic character.

Monte Carlo simulation was employed for forward propagation of the aleatory type input uncertainties

- Gela,
- R. Garga,
- C. Tongd,
- M. Shahnama,
- C. Guenther [5]

The Monte Carlo method can be used to address any mathematical problem or model that is too complex, time consuming, or resource intensive to solve analytically. Instead of tackling the numerical problem directly, Monte Carlo allows the researcher to obtain an approximation of the solution through setting up an experiment of statistical sampling. As the name indicates, the method borrows from games of chance such as those played at the famous casinos of Monte Carlo in Monaco. The Monte Carlo method relies on realizations (draws) from a probability density function. Ideally, to correctly apply the Monte Carlo method and obtain valid results, the sampling method employed should be completely random. The number of realizations has to be sufficiently large to accurately represent the distribution of the input variables. [9]

The computer simulation Monte Carlo is a mathematical-statistical method of risk measurement of individual variants. It is used when there are multiple risk factors that affect the performance of investment projects.

To facilitate the MCMC analysis, a statistical model was generated in order to implement a Bayesian approach. Bayesian modeling draws from two types of knowledge to derive model parameter estimates: prior knowledge, as described in initial parameter distributions, and information that can be deduced from measured data, if appropriately analysed. [2, 5]

Markov chain Monte Carlo (MCMC) methods have been an important algorithm in various scientific fields [8, 10]. MCMC methods can generate samples that follow a target distribution by using a simple proposal distribution. However, in sampling from a complex distribution such as a multimodal one, the standard MCMC methods produce samples that theoretically converge to the target distribution but practically do not. The produced samples can be trapped in a local mode for an extremely long period. [1]

The essence of Monte Carlo simulation is to generate a large number of scenarios and criteria values conversions for each scenario. Simulation outputs can be displayed in numeric or graphical form.

We apply Monte Carlo simulation to company profit in this paper. Profit is an important characteristic determining the success, respectively failure of the business and is mainly used in the methods of profitability determining and business performance. Profit represents trading income, i.e. the difference between revenues and costs, and a strong emphasis is placed on it from the view of all interested groups, stakeholders, both owners and investors, but also creditors whose key interests are free resources on debt settlement. Profit as a target performance indicator is interacted with the market. [7]

One of the Monte Carlo simulation's parts is the sensitivity analysis, which allows calculate the sensitivity of the selected project financial criteria to possible changes in the values of risk factors that affect this criterion. With this analysis we can determine how is the profit, as an evaluation criterion, influenced by the sensitivity of quantity, selling price or cost. Risk factors, where changes in the selected criteria are small are less important, and vice versa. The advantages of sensitivity analysis are simplicity and graphical clearness. On the other hand, the main disadvantage is ignoring the different rates of uncertainty of the individual factors.

Monte Carlo can be divided into several steps, according to Souček. First four steps represent the simulation preparation and the last one simulation performance:

1. Investment project model design and processing in a computer program
2. Identification of key risk factors
3. Determination of likelihood distribution of factors
4. Determination of statistical dependence of risk factors
5. Simulation performance and interpretation of simulation results [11]

By creating the project model it is important to consider the level of detail of the model, which is intended to represent a compromise between simplicity and credence of the project. It is important to recognize the complexity of relationships between profit, as an evaluating criterion, and various risk factors. Souček highlights to keep the principle that all input variables of the model (risk factors) should form a separate, distinct part of the model. [11]

Risk factors represent the input variables of the financial model, which significantly affect the simulation output uncertainty in the form of evaluation criteria. Key risks are those that are sensitive to even the smallest changes in inputs of a simulation and are uncertain. To determine this sensitivity we use the already mentioned sensitivity analysis. It is recommended to start with a larger number of factors and according to simulation results try to minimize them.

2 Practical application of Monte Carlo Simulation

The first step in profit simulation is to define the basic variables affecting the profit. Monte Carlo simulation is applied to an enterprise which operates in the field of mechanical engineering

and industrial engineering and plans to invest in new production hall.

Estimated investment costs are:

€1.9 million with probability 0.28

€2.5 million with probability 0.50

€2.025 million with probability 0.22

The amount of fixed costs is €400 000 and the lifetime is twenty years. Estimated sold quantity is:

1 770 units with likelihood 0.30

1 950 units with likelihood 0.50

2 250 units with likelihood 0.20

Retail price is set on:

€367.5 with likelihood 0.25

€275.0 with likelihood 0.45

€280.0 with likelihood 0.30

Manufacturing overhead costs:

€1.5 with likelihood 0.30

€3.0 with likelihood 0.40

€4.5 with likelihood 0.30

Material costs are:

€3.0 with likelihood 0.35

€6.0 with likelihood 0.45

€7.5 with likelihood 0.20

Unit labor costs are set on:

€30.0 with likelihood 0.28

€33.0 with likelihood 0.42

€40.5 with likelihood 0.30

We used MS Excel to simulate individual fundamentals that influence the result. Each characteristics is simulated on 1 000 repetitions.

Table 1 Investment costs simulation examination, part 1

	A	B	C
1	Investment costs	Likelihood	Cumulative likelihood
2	1 900 000	0.28	0.28
3	2 500 000	0.50	0.78
4	2 025 000	0.22	1.00

Table 2 Investment costs simulation examination, part 2

	D	E
1	Random number	Simulated investment costs
2	=RAND()	
3	=RAND()	
4	=RAND()	

Simulated investment costs are calculated via the function:

$$=IF(\$D2<=\$C\$2;\$A\$2;IF(\$D2<=\$C\$3;\$A\$3;\$A\$4)) \quad (1)$$

Sales volume, unit selling price, unit variable costs consisting of material costs, production overheads and unit wage are simulated in a similar way. Fixed costs are not subjected to the simulation.

The operating result is simulated as the last one. We use following formulas to simulate it:

$$P = I - C \quad (2)$$

$$I = S = q \cdot c \quad (3)$$

$$C = q \sum_{i=1}^n v_i + \sum_{j=1}^m f_j + \frac{I}{S} \quad (4)$$

$$P = q \cdot c - \left[q \sum_{i=1}^n v_i + \sum_{j=1}^m f_j + \frac{I}{S} \right] \quad (5)$$

$$P = q \cdot \left[c - \sum_{i=1}^n v_i \right] - \sum_{j=1}^m f_j - \frac{I}{S} \quad (6)$$

Where:

P – profit

I – incomes

C – costs

S – sales

q – sold quantity

c – retail price

v_i – variable costs

f_j – fixed costs

I – investment costs

T – lifetime

Table 3 Simulated input values – costs, quantity, price

Simulated invest. Costs	Simulated quantity	Simulated retail price	Simulated material costs
1 900 000	1 770	367.5	3
2 500 000	1 950	275	6
2 025 000	2 250	280	7.5

Table 4 Simulated input values – overhead, wages, profit

Simulated manufact. Overhead	Simulated wages	Simulated profit
1.5	30	94 410
3	33	-70 650
4.5	40.5	10 625

Then we determined the most appropriate statistical distribution - Normal Gaussian distribution, which allows us to statistically process the results for the simulated profit. Values of simulated profits are divided into intervals from $-\infty$ to ∞ , the interval range is 15 000.

Table 5 Normal distribution of simulated profit

	A	B	C
1	Lower interval limit	Upper interval limit	Occurrence frequency
2	$-\infty$	- 25 000	1
3	-25 000	0	0
4	0	25 000	0
5	25 000	50 000	0
6	50 000	75 000	15
7	75 000	100 000	92
8	100 000	125 000	139
9	125 000	150 000	202
10	150 000	175 000	154
11	175 000	200 000	40
12	200 000	∞	357

Because we used the Normal distribution we determine the parameters of this distribution, which are subsequently converted. In this case, we chose the following:

- Modus
- Median
- The standard deviation
- The arithmetic mean
- Lower and upper quartile

The arithmetic mean is calculated as follows:

$$\bar{X} = \sum_{i=1}^k x_i \cdot f_i \quad (7)$$

Where:

x_i – interval mean value

f_i – relative occurrence frequency

The mode is the value of symbol X, which occurs in particular empirical set most frequently. In the frequency distribution lines it is that value x_i , which comes with the highest absolute or relative frequency. This value does not have to be represented by the distribution exactly, because multiple values can occur with the same highest frequency. [3]

$$\hat{x} = a + h \cdot \frac{d_0}{d_0 + d_1} \quad (8)$$

Where:

a – lower limit of modal interval

h – interval range

d_0 – the difference between modal and previous interval frequency

d_1 – the difference between modal and next interval frequency

Median is the fair value \tilde{x} , which divides an identified set of values x_1, x_2, \dots, x_n arranged in ascending order of size into two equal-sized parts. Median basic advantage is that it is not affected by the extreme values. [4]

$$\tilde{x} = a + h \cdot \frac{0.5 - F_{\tilde{x}-1}}{f_{\tilde{x}}} \quad (9)$$

Where:

a – lower limit of median interval

h – interval range

$F_{\tilde{x}-1}$ – relative cumulative frequency of previous interval

$f_{\tilde{x}}$ – absolute frequency of median interval

The standard deviation is the square root of the variance and expresses the dispersion of the values around the mean. It represents how these values differ from the mean value. Because the dispersion is calculated in the square units of measure, it cannot be logically interpreted. That is why we try to get a degree of variability, which is expressed in the original units of measurement. [3]

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \cdot p_i} \quad (10)$$

Where:

n – total number of units,

x_i – individual selection units,

p_i – occurrence likelihood of i - event

\bar{x} – arithmetic mean

Lower quartile divides the unit set so that the value of variable x of quarters of units is lower or equal to the first quartile and the value of the variable x of three-quarters of units is higher or equal to the first quartile.

$$\tilde{x} = a + h \cdot \frac{0.25 - F_{\tilde{x}-1}}{f_{\tilde{x}}} \quad (11)$$

$$\tilde{x} = a + h \cdot \frac{0.75 - F_{\tilde{x}-1}}{f_{\tilde{x}}} \quad (12)$$

Where:

a – lower limit of median interval

h – interval range

$F_{\tilde{x}-1}$ – relative cumulative frequency of previous interval

$f_{\tilde{x}}$ – absolute frequency of median interval

The following tables show the calculation of statistical characteristics of Monte Carlo simulation. In addition to absolute frequency detected by a computer simulation, we calculate the relative, absolute cumulative and relative cumulative frequency. The indicator mean was calculated as the conjunction of relative frequency and mean value. The resulting average profit is quantified as the sum of all partial averages. In our case, the average profit is €120 910.

The probability that the company will generate the loss in particular period is 0.1.

Table 6 Simulation result – arithmetic mean, part 1

	C	D	E	F
Class	Absolute frequency	Relative frequency	Cumulative absolute frequency	Cumulative relative frequency
1	1	0,0010	1	0,0010
2	0	0,0000	1	0,0010
3	0	0,0000	1	0,0010
4	0	0,0000	1	0,0010
5	15	0,0150	16	0,0160
6	92	0,0920	108	0,1080
7	139	0,1390	247	0,2470
8	202	0,2020	449	0,4490
9	154	0,1540	603	0,6030
10	40	0,0400	643	0,6430
11	357	0,3570	1000	1,0000
Total	1000	1,0000	x	x

Table 7 Simulation result – arithmetic mean, part 2

	G	H
Class	Mean value	Arithmetic mean
1	-37500,00	-37,50
2	-12500,00	0,00
3	12500,00	0,00
4	37500,00	0,00
5	62500,00	937,50
6	42500,00	3910,00
7	112500,00	15637,50
8	137500,00	27775,00
9	162500,00	25025,00
10	187500,00	7500,00
11	112500,00	40162,50
Total	x	120910,00

Modal interval was defined in Class 11, in interval from 200 000 to ∞ . It was calculated via the formula 4.

$$\tilde{x} = 200000 + 25000 \cdot \frac{(0.357 - 0.04)}{(0.357 - 0.04) + 0.357}$$

The calculation shows that the most frequent value of the simulated profit is **€211 758.16**.

Building on the characteristics of the median as the middle value of the character of a statistical set, median interval is in Class 9.

$$\tilde{x} = 150000 + 25000 \cdot \frac{0.5 - 0.449}{0.154}$$

The mean value of simulated profit was determined on the level of **€158 279.22**.

Lower quartile distributes the statistical set in the ratio of 0.25 to 0.75. Based on this definition, the lower quartile interval is represented by the Class 8.

$$\tilde{x} = 125000 + 25000 \cdot \frac{0.25 - 0.247}{0.2020}$$

With the likelihood of 0.25, the simulated profit will be lower than €125 371.29 and with 0.75 the simulated profit will be higher than €125 371.29.

A similar procedure is applied to determine the upper quartile; its interval will be the Class 11.

$$\tilde{x} = 200000 + 25000 \cdot \frac{0.75 - 0.643}{0.357}$$

With the likelihood of 0.75, the simulated profit will be lower than €207 493.00 and with 0.25 the simulated profit will be higher than €207 493.00.

Table 8 Simulation result – standard deviation

	I	J
Class	Dispersion	Standard deviation
1	3 909 375,63	1 977,21
2	0,00	0,00
3	0,00	0,00
4	0,00	0,00
5	21 065 634,38	4 589,73
6	28 094 557,50	5 300,43
7	1 063 610 711,88	32 613,05
8	2 555 426 376,25	50 551,23
9	2 910 503 846,25	53 949,09
10	1 055 925 025,00	32 495,00
11	2 731 719 598,13	52 265,85
Total	10 370 255 125,00	101 834,45

To determine the extent of variability of simulated profit we used standard deviation and we built on the formula No. 10 The actual profit value may differ from the average value of **± €101 834.45**.

Literature:

1. ARAKI, T., IKEDA, K., *Adaptive Markov chain Monte Carlo for auxiliary variable method and its application to parallel tempering*. Neural Networks 43 (2013). 33-40 p.

2. BERNILLON, P., BOIS, F.Y., *Statistical issues in toxicokinetic modeling: a Bayesian perspective*. Environ. Health Perspect. 2000. 108 (Suppl. 5), 883–893.
3. CISKO, Š., & KLIEŠTIK, T. (2009). *Finančný manažment podniku I*. (1st ed., p. 559). Žilina: EDIS Publishers.
4. CISKO, Š., & KLIEŠTIK, T. (2013). *Finančný manažment podniku II*. (1st ed., p. 769). Žilina: EDIS Publishers.
5. GEL, A., GARG, R., TONG, C., SHAHNAM, M., GUENTHER, C. *Applying uncertainty quantification to multiphase flow computational fluid dynamics*. Powder Technology 242 (2013) 27-39 p.
6. GELMAN, A. *Inference and monitoring convergence*. Richardson, S., Spiegelhalter, D.J. (Eds.), Markov Chain Monte Carlo in Practice. Chapman & Hall/CRC, Boca Raton, p. 131–143
7. KLIEŠTIK, T., BIRTUS, M. (2012). CorporateMetrics as the Methodology Focused on the Measuring of the Market Risk. *Ekonomicko-manažérské spektrum* 6 (1), 80 – 84.
8. LIU, J. Monte Carlo strategies in scientific computing. New York: Springer. 2011.
9. RATICK, S., SCHWARZ, G. *Monte Carlo Simulation*. International Encyclopedia of Human Geography. 2009. 175-184 p.
10. ROBERT, C., & CASELLA, G. Monte Carlo statistical methods. Springer. 2004.
11. SOUČEK, I., FOTR, J. *Podnikatelský zámer a investiční rozhodování*. Grada Publishing, Praha, 2005, ISBN 80-247-0939-2

Acknowledgement

The article is an output of scientific project VEGA 1/0357/11 Kliešтик, T. and col.: Research on the possibility of applying fuzzy-stochastic approach and CorporateMetrics as tools of quantification and diversification of business risk.

Primary Paper Section: A

Secondary Paper Section: AH, BB