PASSIVE PORTFOLIO MANAGEMENT BASED ON QUADRATIC INDEX TRACKING

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Abstract: Strategies of passive management in portfolio selection are based on imitating the performance of a specific benchmark (that is designed in such a way that it approximates as best as possible the value of the market portfolio) with the intention of achieving minimum discrepancy between the benchmark performance and the tracking portfolio performance. In the paper attention is given to portfolio selection based on partial replication of the S&P 500 Index. In contrast to the traditional Markowitzian approach, the key criterion of portfolio selection is minimization of the quadratic tracking error variance. In the empirical exercise, out of the stocks represented in the S&P 500 Index one stock was chosen randomly by each of the 10 GICS sectors and this selection of 10 stocks were available for portfolio selection. On the scale of performance, the quadratic index tracking strategy can be seen superior to the traditional Markowitzian approach.

Keywords: Passive portfolio management. Quadratic index tracking. Tracking portfolio. Tracking error.

1 Introduction

There are various approaches to portfolio selection and two broad classes can be singled out, active asset management and passive asset management. Procedures that are practised under the active approach embrace the well-known Markowitzian approach and are generally well known. On the other hand, passive active management is based on replicating a suitably chosen benchmark, usually a financial index, and under this course, portfolio selection is based on minimizing the discrepancy between the portfolio returns and the benchmark returns. Any strategy aiming at the replication of the underlying financial index is addressed as index tracking. Several approaches to index tracking have been devised and one of them, quadratic index tracking, is expounded in the paper. After a short theoretical outline, the exposited topics are demonstrated in an empirical exercise. In this exercise, four random selections of ten assets represented in the S&P 500 Index are made in a stratified way so that each sector of the GICS classification is represented by one respective asset. For each selection, a total of five portfolios are constructed for the purpose of comparison. This comparison involves juxtaposing the quadratic index tracking strategy, in two variants, to the traditional Markowitzian approach, in two variants, and evaluating the results with respect to the performance of the underlying S&P 500 Index. The empirical exercise is designed so as to highlight and demonstrate some properties of the quadratic index tracking strategy as compared to the traditional Markowitzian approach. Its results are interesting for the investor. All though the quadratic index tracking strategy proves insufficient to guarantee that the benchmarked index is outperformed nor it can be deemed universally preferable than the traditional Markowitzian approach; if performance is defined in terms of the relation between mean return and standard deviation, it eventually delivers higher performance.

Save the introduction and the conclusion, the core of the paper consists of three sections. The section that follows is a short presentation on the traditional Markowitzian approach and is appended by theoretical exposition of the quadratic tracking error in the other section. The final section explains the design of the empirical exercise and shows its results.

1 Markowitzian approach

In the approach of Markowitz, the rational investor balances the expected return and the risk of his portfolio exposition. This may be easily put in the words of Markowitz (1952, p. 79): "There is a rule which implies both that the investor should diversify and that he should maximize expected return. The rule states that the investor does (or should) diversify his funds among all those securities which give maximum expected return." To set up this considerations in practice, some framework is needed. To this end, it is assumed that the portfolio of the investor is to be created of k risky assets. Suppose that these k risky asset returns are represented by a random vector $\boldsymbol{\xi} = (\xi_1, ..., \xi_k)'$ that have an expectation $\boldsymbol{\mu} = (\mu_1, ..., \mu_k)'$ and an $k \times k$ covariance matrix $\boldsymbol{\Sigma} = (\Sigma_{ij})_{k\times k}$ (the diagonal elements Σ_{ii} are variances σ_i^2 of individual returns and non-diagonal elements are respective covariances). Assume for now that the both the expectation $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$ are known. Any portfolio Π with a set of k weights $\boldsymbol{\omega} = (\omega_1, ..., \omega_k)'$ that decide allocation of available financial funds across individual risky assets has expected return $\boldsymbol{\omega}'\boldsymbol{\mu}$ and variance $\boldsymbol{\omega}'\boldsymbol{\Sigma}\,\boldsymbol{\omega}$. If the investor desires the expected return μ_0 (at least), he would pursue a two-step optimization program. In the first step, he would determine the weights of the portfolio with the expectation μ_0 and a minimum variance by solving the following task,

$$\min_{\boldsymbol{\omega}\in\Re^k} \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} \quad \text{subject to} \quad \boldsymbol{\omega}'\mathbf{1} = 1 \quad \& \quad \boldsymbol{\omega}'\boldsymbol{\mu} = \mu_0 \tag{1}$$

Let σ^2_0 denote the variance that is delivered by the optimal solution of the first step. The investor would then see that at the level of risk expressed by σ^2_0 there exists no portfolio whose expectation is possibly higher than the desired expectation μ_0. This would be facilitated by the somewhat reversed optimization,

$$\max_{\boldsymbol{\omega}\in\Re^k} \boldsymbol{\omega}'\boldsymbol{\mu} \quad \text{subject to} \quad \boldsymbol{\omega}'\mathbf{1} = 1 \quad \& \quad \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} = \sigma_0^2. \tag{2}$$

It is frequently the case that this formulation of the optimization task is extended by the short selling restriction.

In practice, however, quantities $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are not known and must be substituted by their respective estimates. Ordinarily, the expectation vector $\boldsymbol{\mu}$ is estimated by simple averaging of individual historical asset returns and an estimate of the covariance matrix $\boldsymbol{\Sigma}$ is produced by a traditional unbiased estimator. Since these estimators are well known and used by default, their description is omitted in the paper. This is called here as the "classical approach". As an alternative to this, these inputs may be estimated by some robust procedure. In the paper, they are estimated by the fast Minimum Covariance Determinant (MCD) estimator proposed by Rousseeuw and van Driessen (1999). This approach is addressed as the "robust (MCD) approach".

2 Quadratic tracking error

No probabilistic framework is made use of here, just a sample of historical observations on returns computed for the benchmark as well as for the risky assets whose portfolio is to mimic the benchmark are employed as an input. In this, assume that a history of T historical observations of logarithmic returns is available and that the tracking portfolio is to be composed of k assets. Let $\mathbf{Y} = (Y_1, ..., Y_T)'$ denote a $(T \times 1)$ vector of benchmark returns, and $\mathbf{X} = (\mathbf{x}_1 \,|\, ... \,|\, \mathbf{x}_T)'$ denote a $(T \times k)$ matrix of returns of the k assets that are to be represented in the tracking portfolio. The symbol $\boldsymbol{\omega}$ stands for a $(k \times 1)$ vector of unknown portfolio weights that are obtained by minimizing the following quadratic optimization problem

$$\min_{\boldsymbol{\omega}\in\Re^k} (\mathbf{Y} - \mathbf{X}\boldsymbol{\omega})'(\mathbf{Y} - \mathbf{X}\boldsymbol{\omega}) \quad \text{subject to} \quad \boldsymbol{\omega}'\mathbf{1} = 1, \tag{3}$$

in which $\mathbf{1}$ is a $(k \times 1)$ vector of ones. This general formulation of the optimization task allows an extension and can be complemented by the constraint banning short sales.

This initial (and somewhat traditional) quadratic programming approach to portfolio tracking is one way to determine the tracking portfolio. Its slight modification gives another possibility of finding (an estimate of) the vector $\boldsymbol{\omega}$. The program in (3) can be restated as an ordinary least squares (OLS) problem. Minimizing the sum of squares of portfolio tracking error deviations is equivalent to running an OLS regression of benchmark returns on individual asset returns to be represented in the tracking portfolio. In this formulation of a regression without intercept, regression coefficients are portfolio weights and residuals form tracking errors.

To this end, the constraint $\boldsymbol{\omega}'\mathbf{1}$ is dropped for a while. Writing a $(T \times 1)$ vector of tracking errors as $\boldsymbol{\varepsilon}$ (with $\boldsymbol{\varepsilon} := \mathbf{Y} - \mathbf{X}\boldsymbol{\omega}$) and assuming that $\mathbf{X}$ is of full column rank, it is evident that the problem can be well expressed via a regression model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\omega} + \boldsymbol{\varepsilon} \tag{4}$$

and that the objective function is minimized for

$$\bar{\boldsymbol{\omega}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}. \tag{5}$$

In order to obtain the final weights $\boldsymbol{\omega}$, the OLS weights $\bar{\boldsymbol{\omega}}$ are then normalized so as to sum to one, i.e.

$$\boldsymbol{\omega} = \bar{\boldsymbol{\omega}} / \bar{\boldsymbol{\omega}}'\mathbf{1}. \tag{6}$$

The quantity $\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}$ is sometimes inaptly called tracking error variance, but it is merely a tracking error measure in quadratic portfolio tracking. In fact, as a quadratic tracking error measure an alternative and interpretationally easier quantity TEM_{QUAD} comparable to a standard deviation might be employed,

$$TEM_{QUAD} = \sqrt{\tfrac{1}{T}\sum_{t=1}^{t=T}(Y_t - \bar{\boldsymbol{\omega}}'\mathbf{x}_t)^2}. \tag{7}$$

This measure is a non-central measure and is thus influenced not only by random positive or negative deviations but also by a possible underperformance or outperformance relative to the benchmark. Estimates produced by estimator (5) are attractive in a statistical sense as this estimator is a best linear unbiased estimator in model (4).

Theoretical details and a further exposition on these issues may be found in Rudolf, Wolter and Zimmermann (1999) or in Prigent (2007).

3 Empirical Exercise

In the empirical exercise, out of the stocks represented in the S&P 500 Index one stock was chosen randomly by each of the 10 GICS sectors and this selection of 10 stocks were available for portfolio selection. This was repeated four times. The four samples of these stocks is displayed in Table 1. It was without design that these samples do not overlap and not stock is repeated in a different sample.

Table 1. The stocks participating in the empirical exercise

GICS sector	SELECTION 1	SELECTION 2	SELECTION 3	SELECTION 4
Consumer Discretionary	Starbucks Corp. (SBUX)	Carnival Corp. (CCL)	Penney (J.C.) (JCP)	Amazon.com Inc (AMZN)
Consumer Staples	Reynolds American Inc. (RAI)	McCormick & Co (MKC)	General Mills (GIS)	Kimberly-Clark (KMB)
Energy	Alpha Natural Resources (ANR)	Occidental Petroleum (OXY)	Noble Corp (NE)	Marathon Oil Corp. (MRO)
Financial	SLM Corporation (SLM)	Berkshire Hathaway (BKR.B)	Vornado Realty Trust (VNO)	Regions Financial Corp. (RF)
Health Care	Merck & Co. (MRK)	BIOGEN IDEC Inc (BIIB)	Cardinal Health Inc. (CAH)	Allergan Inc (AGN)
Industrials	Joy Global Inc. (JOY)	Varian Medical Systems (VAR)	General Electric (GE)	CSX Corp (CSX)
Information Technology	Teradata Corp. (TDC)	Akamai Technologies Inc (AKAM)	TE Connectivity Ltd. (TEL)	Xilinx Inc (XLNX)
Materials	Vulcan Materials (VMC)	Allegheny Technologies Inc (ATI)	Alcoa Inc (AA)	The Mosaic Company (MOS)
Telecommunications Services	AT&T Inc. (T)	Frontier Communications (FTR)	Crown Castle International Corp (CCI)	Frontier Communications (FTR)
Utilities	EQT Corporation (EQT)	Sempra Energy (SRE)	NRG Energy (NRG)	Consolidated Edison (ED)

Source: the authors.

In computations and preparing graphical presentations, the software R version 3.0.1 (R Core Team, 2013) was employed with several of its libraries, `quadprog` (Turlach and Weingessel, 2013), `timeSeries` (Wuertz and Chalabi, 2013), `PerformanceAnalytics` (Carl et al., 2013), `fPortfolio` (Rmetrics Core Team and Wuertz, 2011) and `FRBData` (Takanayagi, 2011).

In the empirical exercise the in-sample-period ran from 1 Jan 2011 to 31 Dec 2012 and the out-of-sample period stretched from 1 Jan 2013 to 20 Nov 2013. Whereas the in-sample period was made up of 500 effective logarithmic returns on a trading daily basis, there were 225 effective logarithmic returns present in the out-of-sample period. Returns for the in-sample period were an ingredient in estimating the parameters of the Markowitzian mean-variance optimization (the expectation vector as well as the covariance matrix of returns) in a classic non-robust way as well as in a robust fashion (in which the MCD estimator was of use). These estimates were instrumental in alternative Markowitzian portfolio selection with the desired expected return equivalent to the average in-sample return of the S&P 500 Index, i.e. 0.023 % p.d. (equivalent to 5.71 % p.a.). In their construction, short sales were permitted. The behaviour of these two Markowitzian portfolios is compared to the behaviour of two tracking portfolios, one of which is constructed by means of the normalized OLS estimator in (6), and the other through the constrained quadratic optimization in (3). These portfolios were tracked with respect to the S&P 500 Index, and in their construction, returns of the in-sample period were of use. The weights of the four optimized portfolios are shown in Table 2. Note that – despite the fact that no restriction on short sales was placed in portfolio tracking – all the tracking portfolios consists of long positions only. The composition of the Markowitzian portfolios in individual samples is not mutually dissimilar.

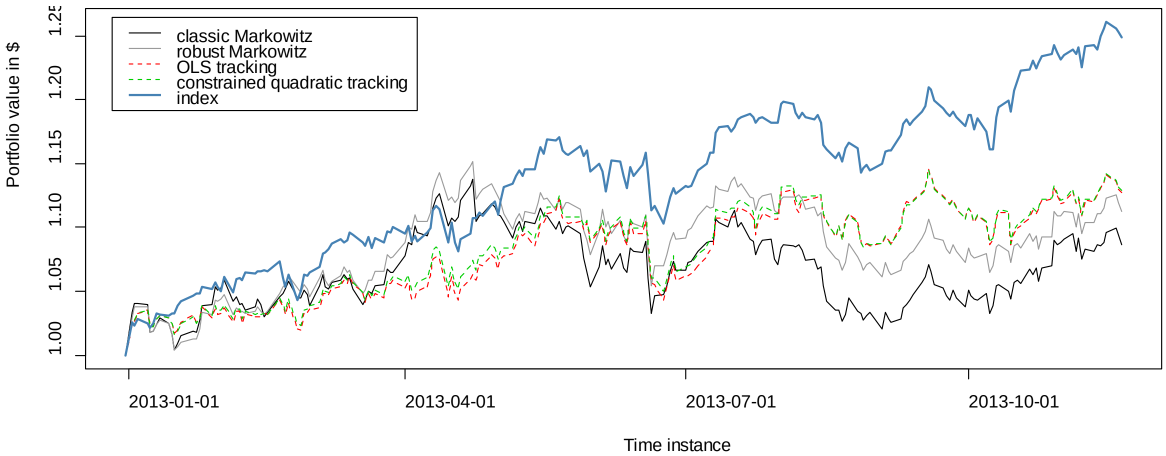
Table 2. The composition of individual portfolios

SAMPLE 1	**SBUX**	**RAI**	**ANR**	**SLM**	**MRK**	**JOY**	**TDC**	**VMC**	**T**	**EQT**
Classical Markowitz	2.89%	35.66%	2.02%	-13.48%	23.17%	-1.29%	-0.69%	-1.99%	56.52%	-2.80%
Robust (MCD) Markowitz	2.66%	32.30%	2.94%	-4.80%	27.61%	-2.11%	1.40%	-8.78%	50.39%	-1.59%
OLS quadratic tracking	8.17%	12.09%	2.49%	9.53%	15.91%	9.15%	8.53%	4.05%	21.79%	8.30%
Constr. quadratic tracking	7.85%	14.76%	1.85%	7.30%	16.46%	7.82%	7.69%	3.44%	25.27%	7.56%
SAMPLE 2	**CCL**	**MKC**	**OXY**	**BRK.B**	**BIIB**	**VAR**	**AKAM**	**ATI**	**FTR**	**SRE**
Classical Markowitz	4.91%	53.03%	-3.19%	1.92%	-6.24%	1.31%	1.09%	-5.45%	17.07%	35.56%
Robust (MCD) Markowitz	-0.80%	35.69%	3.09%	17.80%	-6.04%	11.35%	2.33%	-9.14%	13.92%	31.79%
OLS quadratic tracking	7.73%	13.37%	14.84%	25.49%	5.09%	6.51%	2.46%	6.83%	1.92%	15.77%
Constr. quadratic tracking	6.70%	18.43%	12.17%	22.77%	5.48%	5.75%	2.26%	4.83%	2.37%	19.24%
SAMPLE 3	**JCP**	**GIS**	**NE**	**VNO**	**CAH**	**GE**	**TEL**	**AA**	**CCI**	**NRG**
Classical Markowitz	3.05%	76.57%	-4.09%	6.53%	17.55%	-6.85%	0.76%	0.09%	6.79%	-0.40%
Robust (MCD) Markowitz	2.63%	74.33%	2.54%	9.38%	16.48%	-14.56%	-6.10%	2.68%	13.28%	-0.66%
OLS quadratic tracking	1.53%	11.69%	6.14%	15.92%	11.12%	18.40%	12.17%	12.17%	7.32%	3.54%
Constr. quadratic tracking	1.44%	19.45%	5.21%	14.17%	11.76%	16.04%	10.81%	8.95%	8.84%	3.34%
SAMPLE 4	**AMZN**	**KMB**	**MRO**	**RF**	**AGN**	**CSX**	**XLNX**	**MOS**	**FTR**	**ED**
Classical Markowitz	1.77%	44.75%	-0.09%	-10.08%	-2.24%	1.35%	5.73%	3.70%	6.92%	48.19%
Robust (MCD) Markowitz	0.71%	52.73%	-5.43%	-8.42%	-1.82%	5.96%	3.73%	3.44%	3.44%	45.67%
OLS quadratic tracking	6.42%	17.80%	4.27%	12.41%	15.67%	0.45%	14.71%	8.74%	2.28%	17.25%
Constr. quadratic tracking	6.27%	19.10%	4.09%	11.59%	15.11%	0.44%	14.44%	8.47%	2.25%	18.24%

Source: the authors.

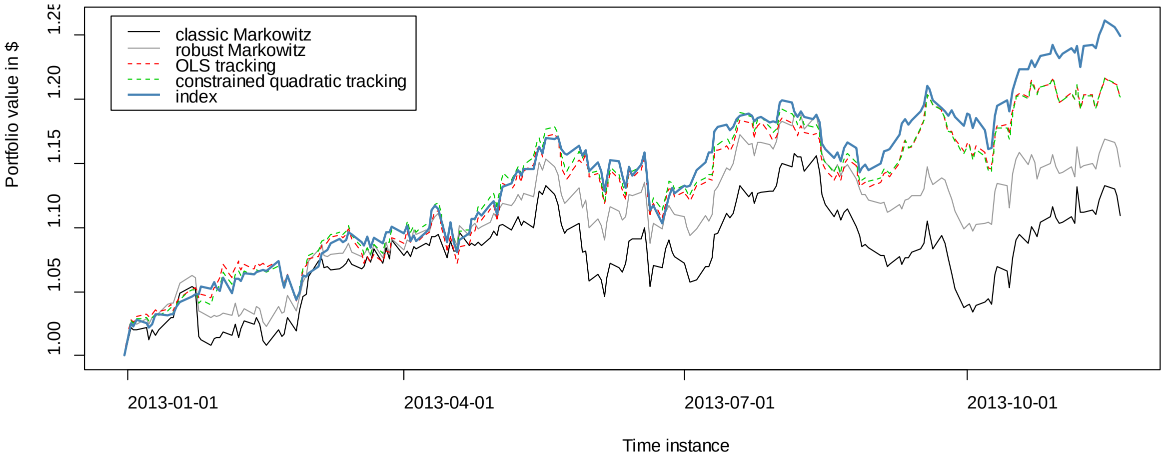
According to the weights reported in Table 2, for each sample, four portfolios were fictively created as of 31 Dec 2012, the last day of the in-sample period, at their initial value $ 1. Simultaneously, on that specific day $ 1 was also fictitiously invested in the S&P 500 Index. The evolution of these portfolio values over the out-of-sample period is shown in Figures 1 to 4.
The out-of-sample behaviour of the four constructed portfolios for each sample can be looked on and interpreted from several perspectives, but from the standpoint of an investor, the most decisive is the value of these portfolios on the last day of the out-of-sample period, as of 20 Nov 2013. No portfolio in any sample delivered a higher value than the S&P 500 Index did. In samples 1, 2 and 4 the ultimate value of index tracking portfolios was as of 20 Nov 2013 higher than the value of Markowitzian portfolios. Only in sample 3 the value of the portfolio constructed by the classic Markowitzian approach was higher than the value of both index tracking portfolios. Having claimed superiority of quadratic index tracking compared to the Markowitzian approach in terms of the final evaluation, it must be said that this is chiefly due to the fact that 20 Nov 2013 was selected as the decisive landmark. If a different day had been chosen for reference, the final impression might have differed.
The evolution of index tracking portfolios was mostly very similar, which was manifested by a very close overlapping or closeness of index tracking portfolio values in samples 1, 2 and 4. A slight discrepancy in the development of index tracking portfolio values is detected only in sample 3. Notable differences were found with the evolution of Markowitzian portfolios in all the samples except sample 4.

Figure 1 The evolution of the value of constructed portfolios (starting on 2012-12-31 at $ 1) over the out-of-sample period for Sample 1



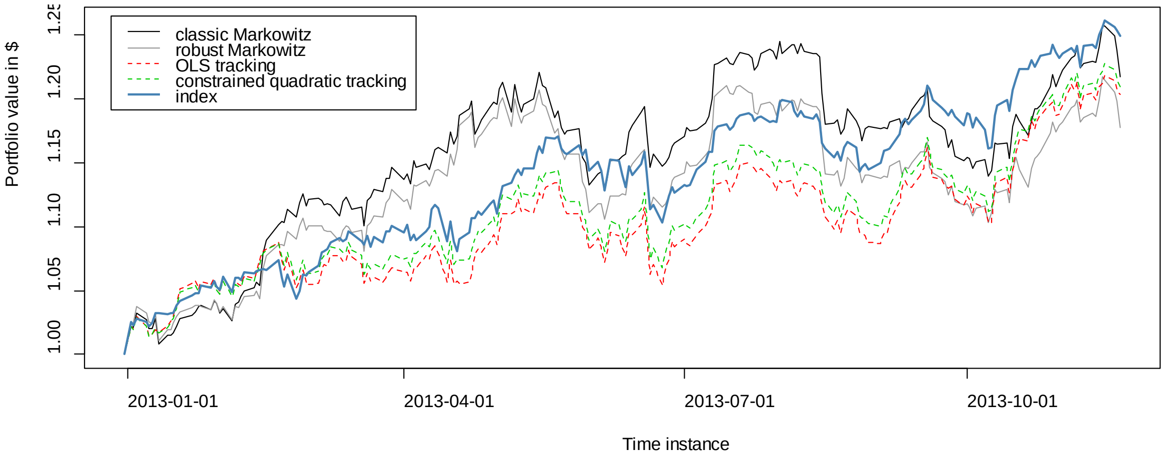
Source: the authors.

Figure 2 The evolution of the value of constructed portfolios (starting on 2012-12-31 at $ 1) over the out-of-sample period for Sample 2



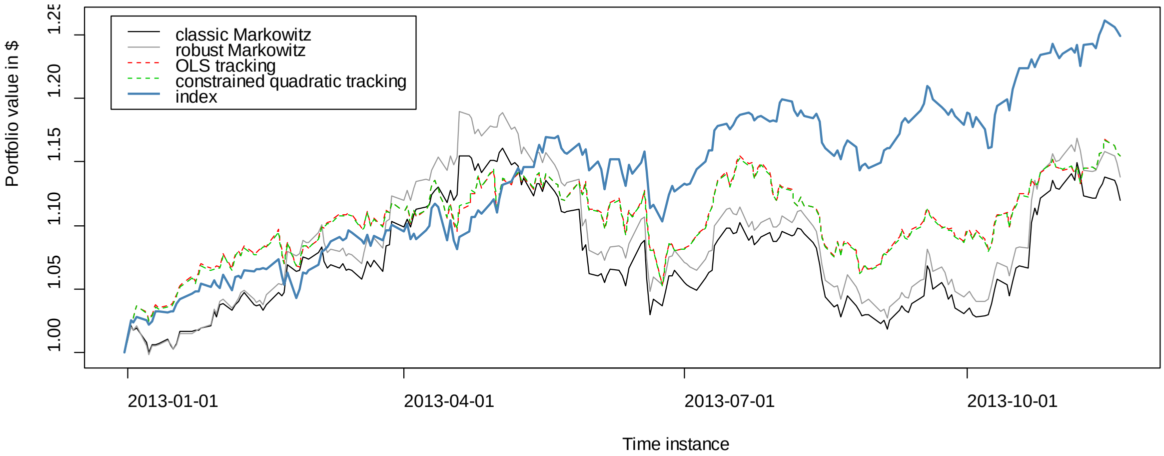
Source: the authors.

Figure 3 The evolution of the value of constructed portfolios (starting on 2012-12-31 at $ 1) over the out-of-sample period for Sample 3



Source: the authors.

Figure 4 The evolution of the value of constructed portfolios (starting on 2012-12-31 at $ 1) over the out-of-sample period for Sample 4



Source: the authors.

The performance of individual portfolios both in the in-sample period and in the out-of-sample period was evaluated by the conventionally used Sharpe ratio (in which computation, the 1Y nominal interest rate on U.S. government securities was used). Individual Sharpe ratios, average returns and risks measured by the standard deviation are reported in Table 3 that follows.

There are some distinctive features in the performance of individual portfolio selection methods, and on this attention is called to the bolded Sharpe ratio values in Table 3. Whilst the Markowitzian portfolios in each sample yielded highest in-sample performance in terms of mean return to standard deviation (also one index tracking portfolio in sample 1 can be tagged in this fashion), in the out-of-sample period it was the index tracking portfolios that showed highest performance on a comparative basis and clearly outperformed both Markowitzian portfolios (with the exception of the classic Markowitzian portfolio in sample 3). Nonetheless, none of the portfolios created indicated a higher Sharpe ratio than the S&P 500 Index.

Table 3. Return characteristics of selected portfolios on a daily basis

	Portfolio selection method	**In-sample indicators** (Jan 2011 – Dec 2012)			**Out-of sample indicators** (Jan 2013 – Nov 2013)		
		Average return (p.d.)	**Standard deviation (p.d.)**	**Sharpe ratio (p.d.)**	**Average return (p.d.)**	**Standard deviation (p.d.)**	**Sharpe ratio (p.d.)**
Sample 1	**Classical Markowitz**	0.02%	0.87%	**0.0256**	0.04%	0.86%	0.0424
	Robust (MCD) Markowitz	0.02%	0.90%	**0.0248**	0.05%	0.81%	0.0584
	OLS quadratic tracking	0.03%	1.27%	0.0196	0.05%	0.79%	**0.0674**
	Constr. quadratic tracking	0.03%	1.20%	**0.0232**	0.05%	0.77%	**0.0696**
	(Investing into) index	0.02%	1.19%	0.0189	0.10%	0.71%	0.1378
Sample 2	**Classical Markowitz**	0.02%	0.97%	**0.0232**	0.05%	0.87%	0.0525
	Robust (MCD) Markowitz	0.02%	1.00%	**0.0224**	0.06%	0.80%	0.0759
	OLS quadratic tracking	0.01%	1.30%	0.0055	0.08%	0.80%	**0.1022**
	Constr. quadratic tracking	0.02%	1.22%	0.0132	0.08%	0.79%	**0.1035**
	(Investing into) index	0.02%	1.19%	0.0189	0.10%	0.71%	0.1378
Sample 3	**Classical Markowitz**	0.02%	0.84%	**0.0267**	0.09%	0.84%	**0.1041**
	Robust (MCD) Markowitz	0.02%	0.85%	**0.0262**	0.07%	0.86%	0.0840
	OLS quadratic tracking	0.00%	1.31%	- 0.0004	0.08%	0.81%	**0.1016**
	Constr. quadratic tracking	0.01%	1.21%	0.0054	0.08%	0.77%	**0.1089**
	(Investing into) index	0.02%	1.19%	0.0189	0.10%	0.71%	0.1378
Sample 4	**Classical Markowitz**	0.02%	0.71%	**0.0313**	0.05%	0.81%	0.0611
	Robust (MCD) Markowitz	0.02%	0.77%	**0.0290**	0.06%	0.84%	0.0680
	OLS quadratic tracking	0.02%	1.16%	0.0177	0.06%	0.82%	**0.0772**
	Constr. quadratic tracking	0.02%	1.14%	0.0188	0.06%	0.82%	**0.0779**
	(Investing into) index	0.02%	1.19%	0.0189	0.10%	0.71%	0.1378

Source: the authors.

Table 3 manifests another property of the approaches: for each of the samples, no matter whether in the in-sample period or in the out-of-sample period, variability of returns found with the portfolios constructed by means of constrained quadratic tracking is lower than with those constructed by means of the unconstrained OLS quadratic tracking. This is further shown in higher values of the Sharpe ratio for the constrained quadratic tracking method. This might also be interpreted as an advantage of constrained quadratic tracking over unconstrained OLS quadratic tracking as the former method delivers more compact portfolios than the latter (of course, compact in terms of variability of portfolio returns). Moreover, it seems that the classical Markowitzian approach generated better results in the in-sample period whilst the robust Markowitzian approach seems preferable in the out-sample period.

4 Conclusion

The paper concentrates on portfolio optimization based on minimizing quadratic tracking error. Under this method to portfolio selection, the aim is to replicate a suitably chosen financial index so that the selected portfolio is as close as possible to the index with respect to the quadratic tracking error. This explains the address of the approach, quadratic index tracking After a brief introduction of the methodology, an empirical exercise was undertaken so as to ascertain behaviour of this method in comparison to the traditional Markowitzian approach. A total of four random samples from stocks represented in the S&P 500 Index were considered and for each sample four portfolios were independently constructed replicating the S&P 500 Index. Two methods were quadratic tracking methods (the unrestricted OLS quadratic tracking and the restricted quadratic tracking) and two methods were traditional Markowitzian methods (the method using non-robust inputs and its robust alternative). Constrained quadratic tracking produce portfolios with lover variability than unconstrained OLS tracking, which leads to higher Sharpe ratios. The results of the empirical study conducted do not permit unequivocally to state as to whether the traditional Markowitzian approach or the tracking error approach is preferable. Some distortions may be caused by the fact that both of these approaches neglect information on long-term trends encoded in asset prices, which may be seen as a deficiency of the methods. Newer approaches are founded on the concept of cointegration. Cointegration is a statistical property of time series that permits modelling non-stationary asset prices instead of stationary asset returns. As financial asset prices are non-stationary with a long memory, the cointegration analysis can be utilized to uncover the complex information in their trends and to explain more aptly their behaviour in the long run. This awareness stimulates further interest with the authors who will focus upon portfolio tracking based on the cointegration analysis in their future research.

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