INVESTMENT DECISION MAKING BASED ON THE SIMULATION METHOD

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Abstract: In preparing financial plans as part of the investment decision making process it is necessary to integrate risk and uncertainty resulting from unpredictability of key factors of investment planning. The paper is devoted to applications of the Monte Carlo simulation method as one of tools that explicitly allow for risk in the investment decision making process. The simulation method provides an interval of possible outcomes for the key factor that shapes the decision making (such as the net present value) and measures risk of the investment project on the basis of an at-risk measure. Usage of the simulation method is clarified by dint of a model and real example in which a decision about feasibility of the investment project is made.

Key words: Net present value. Monte Carlo simulations. Risk factors. Probability distribution.

1 Introduction

Deciding on capital investments as well as capital project implementations requires a thorough comparison of expenditure associated with, and revenue generated by, the respective investment or project. One of the basic criteria that is utilized throughout financial theory and practice in assessing the adequacy and profitability of a project is the criterion of net present value (NPV). NPV is defined as the surplus of discounted revenue generated by the project over discounted expenditure spent for the project, whilst discounting is done so as to reflect the desired rate of return. Expressing NPV as a single number does not permit to capture risk stemming from uncertainty in the future development of key factors that influence the very figure embedded in NPV. Of appropriate use in investment project decisions is not only the knowledge of NPV that fully mirrors anticipations and estimates that are formed prior to the implementation of projects but of great import is also the knowledge of risk that is inherent in NPV. One of the approaches to measuring risk of the project and evaluating sensitivity of the project outcomes is founded on the Monte Carlo simulation method, whose practical application and demonstration is the content of this paper. Facilitated by a case study, the paper exposits the idea of Monte Carlo simulations and their usability in deciding on a project as a sequence of four steps: (i.) mapping of cash flows through a mathematical model into the criterion of NPV, (ii.) identification of risk factors and determination of their probability distribution, (iii.) Monte Carlo simulation of possible outcomes, and eventually (iv.) computation of risk measures tied with the project, on which basis riskiness of the project can be responsibly judged and the decision on acceptation or rejection of the project made.

The paper is organized into three core sections, neglecting the obligatory introduction and conclusion. Whilst, in the ensuing section, the procedure of the Monte Carlo method is outlined; the other two sections expound on the details and results of the case study that is included in the paper.

2 Monte Carlo simulations

There are several approaches to quantification of factors influencing the success of a project, and these in some manner depict uncertainty of future events. To this end, of relatively frequent use is the simple method of sensitivity analysis. Sensitivity analysis seeks to measure sensitivity of the chosen criterion under various scenarios in the development of risk factors. These scenarios are formulated in order to capture different states of the world and include usually a pessimistic variant, a (most probable) medium variant as well as an optimistic variant of the future development of risk factors. This method is fairly simple but it associates itself with ambiguity in the understanding of (the likelihood of) the pessimistic or optimistic variant and another its shortcoming is that it neglects possible dependencies of risk factors and does not attempt to model a different level of uncertainties that are inherent in them (cf. Hnilica and Fotr, 2009, p. 34). The way how sensitivity analysis results are handled is, as a matter of fact, in the hands of the corporate managers, being his subjective choice, and their interpretation crucially depends on his risk preferences (cf. Cisko and Klieštik, 2013, p. 80).

These drawbacks are meliorated to great extent by the Monte Carlo simulation method. Under this method of simulation, a large number of potential scenarios are generated for individual risk factors respecting correlations or relationships in their development. For each scenario, the value of the target criterion is determined and all simulated values are a good approximation of its probability distribution provided the number of Monte Carlo simulation is sufficiently large. The Monte Carlo method requires specification of the marginal probability distribution of individual risk factors and their dependency structure but makes not attempt at expressing their joint probability distribution analytically. The construction of the joint probability distribution of risk factors is substituted by the generation of pseudo-random draws from it (naturally, in the way that the marginal distributions together with the dependence structure are accounted for). Each draw represents a potential scenario of the future state of the world and this interpretation makes it sensible that the desired criterion is evaluated with respect to each scenario so as to arrive at the resulting outcome under the given scenario. Having a sufficiently high collection of these potential outcomes is equivalent to having the underlying probability distribution of the target criterion. In the setting up of this underlying probability distribution, non-parametric methods of statistical analysis are employed and demanded theoretical quantities (such as quantiles or moment characteristics) are computed empirically. Examples of risk measures that are computed in this fashion include standard deviation, value at risk (or other "at risk" equivalent), expected shortfall. To make this procedure workable, it is necessary both to specify the marginal probability distribution of individual risk factors and estimate its parameters, and then to describe duly the dependence structure between parameters and to estimate parameters that are the result of such a description.

Respecting the exposition by Hnilica and Fotr (2009, p. 71), the methodological procedure of Monte Carlo simulations can be structured in these few steps:

- the construction of a mathematical model, through which the target criterion is expressed as a functional of various (deterministic or stochastic) input variates,
- the determination of key risk factors, i.e. input variables that significantly influence uncertainty of the target criterion and determine its value,
- the determination of probability distribution of risk factors,
- the definition of dependence structure between risk factors, possibly through an ancillary statistical model,
- the very process of simulation,
- the graphical and numerical evaluation and presentation of the achieved results.

This procedure is clarified by means of a case study whose basic inputs are borrowed from Brealey et al. (2008, pp. 271-283).

3 Outline of the case study

Brealey et al. (2008, ibid.) in their publication consider an investment project of the Otobai Company in Osaka, Japan on launching of an electrically powered motor scooter for city use. The project requires initial investment of ¥ 15 mds. and is scheduled for the following 5 years with no terminal value at the end of Year 10. It is assumed that revenue for each of Years 1 – 10 are determined by the market size (measured by the number of electric scooters sold), the market share of the Japanese company and by the selling unit price, which is alongside the market size and the market share an exogenous variable difficult to influence for the company. The market size and the market share determine the number the number of scooters sold by the company. Besides the number of scooters sold, cost of the project is each year influenced by variable cost as well as fixed cost, the extent of which is affected by the current situation at the market. Annual fixed cost also comprises the depreciation of the initial investment calculated over the 10-year period on a straight line basis. Annual profit of the project can be represented by the equation

$$\text{profit} = \text{market size} \times \text{market share} \times \times (\text{unit selling price} - \text{unit variable cost}) - \text{fixed cost} \quad (1)$$

is taxed at a rate of 50 %, which is predicted to remain stable for the entire 10-year period. Under these considerations, cash-flows associated with the project can be annually described by the relationship

$$\text{cash-flow} = \text{protit} \times 0.50 + \text{depreciation} \quad (2)$$

It is operated with the 10 percent return of the capital (at the discrete model of compounding of interest), and therefore the criterion of NPV (expressed in ¥) is given by

$$\text{net present value} = -\text{¥}\,15\,000\,000\,000 + \sum_{t=1}^{t=10} \frac{\text{cash-flow in year } t}{1.10^{t}} \quad (3)$$

A thorough analysis of factors affecting NPV in (3) yields that the source of risk imposing uncertainty upon NPV calculations are the market size, the market share, the unit variable cost as well as the fixed cost (without depreciation charges that are known for each year and represent one tenth of the initial investment expense). All these factors are exogenous to the company and can be clearly modelled as random variables with certain probability distribution.

The budgeted (and expected) values of the 5 risk factors considered by the company are reported in Table 1. With these values of risk factors, the project generates annual pre-tax profit

$$1\,000\,000 \times 0.10 \times (375\,000 - 350\,000) - 3\,000\,000 - 1\,500\,000 = \text{¥}\,3\,000\,000\,000,$$

and annual cash-flow in the amount

$$3\,000\,000 \times 0.50 + 1\,500\,000 = \text{¥}\,3\,000\,000\,000,$$

which implies the following calculation of NPV

$$\text{net present value} = -15\,000\,000\,000 + \sum_{t=1}^{t=10} \frac{3\,000\,000\,000}{1.10^{t}} \doteq \text{¥}\ 3\,433\,701\,000.$$

Table 1. Budgeted (and expected) values of risk factors for each year of the project duration

Risk factor	Expected value
Market size (number of electric scooters)	1 000 000
Market share	0.10
Unit selling price of an electric scooter	¥ 375 000
Unit variable costs	¥ 300 000
Fixed cost (non inclusive depreciation)	¥ 3 000 000 000

Source: the authors.

In addition to the budgeted estimates, the original authors of this case study take into account various scenarios for the evolution of risk factors such as the optimistic model or the pessimistic variant, see Brealey et al. (2008, p. 273) and their basis evaluate credibility of the calculated NPV ¥ 3.433 mds. This approach of subjective stress testing is characteristic of certain drawbacks and 3 crucial objection can be raised thereto (cf Boďa and Gavliak, 2007, p. 30): (1.) Scenarios are chosen on a subjective liking and their usability heavily depends on the experience of the user. (2.) It is not possible meaningfully assess individual variants of the future radical development, in which consequence it is of question which of the scenarios is more probable and should be considered in preference. (3:) Frequently it is beyond possibilities to evaluate the completeness or likelihood of produced information in a probabilistic or statistical sense.

Brealey et al. (2008, pp. 278-282) contemplate using Monte Carlo simulations in the description of basic risk properties of the project. Thereon they assume the Gaussian distribution for each risk factor under consideration and they rely on independence of risk factors, which facilitates the construction of the probability distribution of project cash-flows on the basis of 10 000 simulations. They only outline that for each simulated scenario a new figure of NPV should be calculated and the final results should be evaluated.

The paper offers, in the scope of the stated case study on electric scooters and their production, a more complex view on assessing risk of investment projects. The approach indicated by the originators of the case study may be extended by considering more suitable and more typical probability distributions for the identified risk factors, by allowing a higher number of simulations in order to enhance precision of results and by producing a detailed analysis of simulation results. Besides visual and descriptive statistical processing of simulated project NPVs, additional information should be uncovered and identified as to

- under which value the true NPV of the project should not fall at a pre-specified and sufficiently high confidence level (such as 0.95),
- with what probability the project achieves a negative NPV,
- with what probability the true NPV of the project is lower than the budgeted NPV, and
- what relationship exists between risk and return of the project.

4 Simulation setting and interpretation of results

There are only four risk factors under advisement in the contribution, viz. the market size, the market share, the unit selling price and the unit fixed cost. The fixed cost are assumed to be under control of the company (for example because they are contracted beforehand) and shall be in the anticipated amount of ¥ 3 mds. (without depreciation) and ¥ 4.5 mds. with depreciation charges. The combination of these four risk factors determines the amount of cash-flow of the project in Year 1 and it is assumed that cash-flows in the following nine years are not constant but change with respect to the amount of cash-flows in the previous year. In Year 1, the cash-flow of the project is represented by the equation

$$Z = \eta \times \xi \times (\psi - \zeta) - 4\,500\,000\,000$$
$$Y_1 = Z \times 0.50 + 1\,500\,000\,000 \quad (4)$$

in which all variables relate to Year 1: Y_1 is cash-flow in ¥, η stands for the market size (expressed as the number of scooters sold), ξ represents the market share (expressed as percentage), ψ denotes the selling price of scooters in ¥, and, finally, ζ represents unit variable cost in ¥. Variables η, ξ, ψ and ζ are random variates, in which consequence Y_1 is of a stochastic nature as well. The assumed probability distribution of these variables is displayed in Table 2. The selling price of electric scooters and the unit variable cost are modelled in such a way that their amount does not drop beneath a certain level (i.e. situations when the selling price would go under ¥ 250 000 or the unit variable cost would recede under ¥ 200 000 are excluded from possibilities). For the sake of simplicity, the random variates η, ξ, ψ a ζ are treated as independent. It is evident that this assumption is not justified for some pairs of these random variates (e.g. the market size and the market price) or otherwise oversimplifying (such as the market size and the selling price of electric scooters).

Another assumption is that cash-flows in Years 2 – 10 evolve according to an AR(1) process, whilst their expected value is Y_1 (or rather the realized value of Y_1) and the correlation coefficient between cash-flows of two successive years is 0.90. The AR(1) model with Gaussian innovations that is in compliance with these requirements is specified by the equation holding for $t \in \{2,...,10\}$

$$Y_t = 0.10 \times Y_1 + 0.90 \times Y_{t-1} + \varepsilon_t \qquad (5)$$

wherein Y_t denotes the amount of cash-flows in Year t and ε_t is a Gaussian white noise for Year t with zero mean and dispersion 100 000 000^2. The parameters of equation (5) warrants that the expected value of cash-flows in Years 2 – 10 is Y_1 (or rather the realized value of Y_1) and the correlation coefficient between cash-flows between two consecutive years is exactly 0.90.

Table 2. Risk factors of the project and their probability distribution

Risk factor	Distribution	Expected value
Market size (the number of electric scooters)	Normal(1 000 000, 50 000^2)	1 000 000
Market share	βeta(25, 225)	0.10
Selling price of electric scooters	250 000 + LogNormal(11.73107, 0.10^2)	¥ 375 000
Unit variable cost	200 000 + LogNormal(11.50688, 0.11^2)	¥ 300 000
Fixed cost (without depreciation)	constant	¥ 3 000 000 000

Source: the authors.

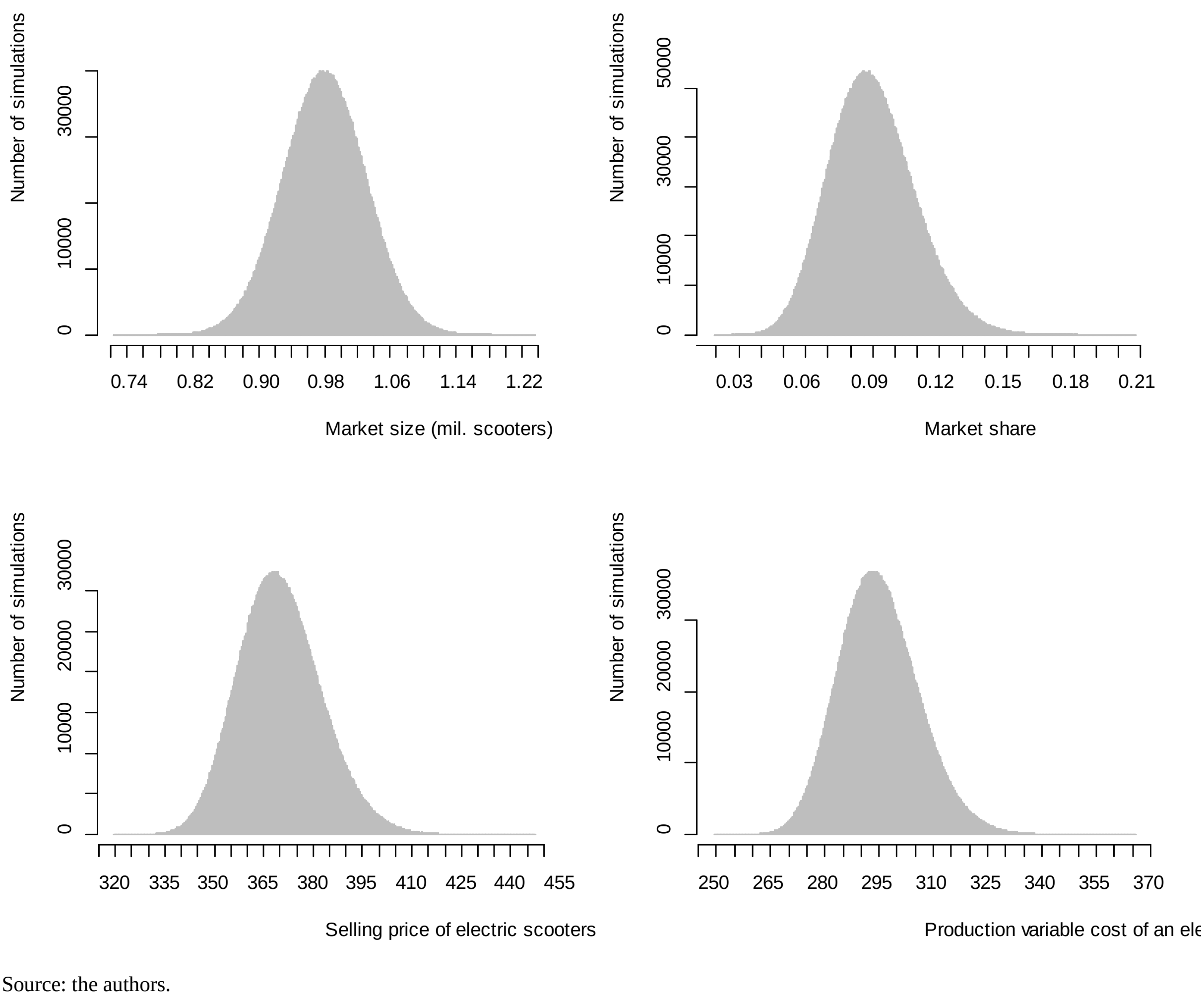
For each simulation, NPV is a function of 13 random variates η, ξ, ψ, ζ, ε_2, ..., ε_{10} , or of their realizations, and the following formula applies

$$NPV(\eta, \xi, \psi, \zeta, \varepsilon_2, \ldots, \varepsilon_{10}) = -15\ 000\ 000\ 000 + \sum_{t=1}^{t=10} \frac{Y_t}{1.10^t} \qquad (6)$$

On the basis of the accomplished mapping of risk factors into the criterion of NPV were under the setting of Table 2 run a total of 5 000 000 Monte Carlo simulations of the variates η, ξ, ψ a ζ in relationship (4) and then produced 5 000 000 Monte Carlo simulation of the future cash-flow development under relationship (5). These scenarios enabled a construction of the empirical distribution of NPV. The number of simulations was derived from the fact that in simplest designs one risk factor requires running 10 000 simulations. Here a higher count of simulations corresponds with the higher number of risk factors. Simulations were run and graphical presentations prepared in program R, version 3.0.1 (R Core Team, 2013). Therein, three specialized libraries of program R were employed, `MASS` (Ripley, Venables et al., 2013), `tseries` (Trapletti and Hornik, 2013), `PerformanceAnalytics` (Carl et al., 2013).

Figure 1 displays the histogram-like shapes of simulated values of the risk factors η, ξ, ψ a ζ.

Figure 1. Empirical distribution of simulated risk factors

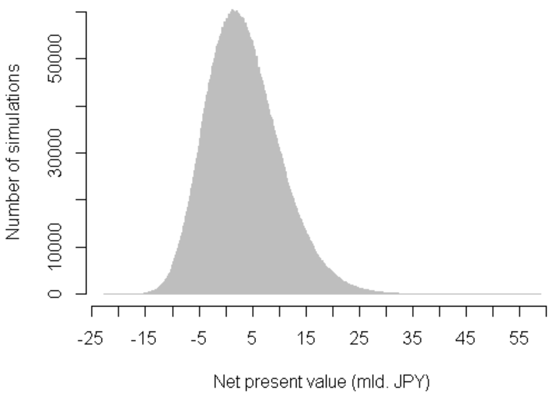


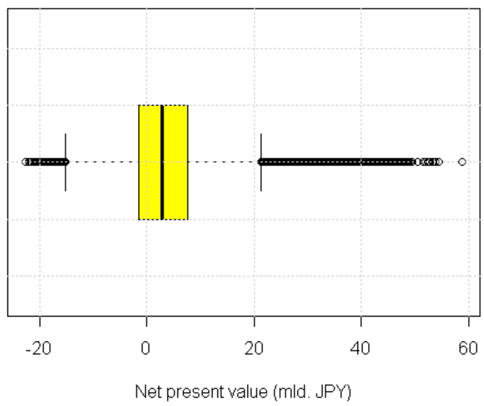
Source: the authors.

Figure 2 shows the final histogram of the NPV of the project as well as the box-plot of simulated values.

As evident from the histogram and the box-plot of simulated values of NPV, the distribution of NPV has a longer right tail and the mass of the distribution is concentrated to the left, which implies that NPV tends to realize values under the expected mean value. In addition, the distribution of NPV is prone to unsatisfactory negative values and leptokurtosis is manifest (owing to the occurrence of outlying and extreme values). In spite of (optically) slight left-tailedness NPV takes preferably high extreme positive values than extreme negative values. The distribution of the NPV of the project is with respect to its asymmetry and leptokurtosis does not reveal compliance with the Gaussian distribution, which is supported by the Jarque-Bera test as well (the Jarque-Bera statistic is 319418.9, the associated p-value is zero).

Figure. 2. Empirical distribution and box-plot of NPV





Source: the authors.

Basic descriptive characteristics of the simulated distribution of NPV confirm the declared properties formulated on the basis of visual inspection and graphical analysis. They are summarized in Table 3.

Table 3. Elementary empirical characteristics of the distribution of NPV

Descriptive	Value
Minimum	¥ -22.438 mds.
Lower quartile	¥ -14.782 mds.
Median	¥ 2.812 mds.
Upper quartile	¥ 7.667 mds.
Maximum	¥ 58.184 mds.
Average	¥ 3.434 mds.
Standard deviation	¥ 6.946 mds.
Coefficient of variation	2.023
Moment skewness	0.551
Moment kurtosis (symmetrized about 0)	0.565

Source: the authors.

The mean NPV is ¥ 3.434 mds. and is in consistence with the expectations on the value of risk factors as systemized in Table 1 and Table 2. The risk of the project can be measured and assessed in several ways.

1. The standard deviation ¥ 6.946 mds. in relation to the expected NPV ¥ 3.434 mds. (representing 202.30 %) indicates high dispersion of true (or rather simulated) values about the (computed) expected value, and in effect is also indicative of high risk as well. Measuring risk by means of the standard deviation nowadays represents a classic approach.
2. A more modern approach rests in measuring risk by “at risk” measures, i.e. downside risk measures, which communicate the worst value of the target criterion over a given time horizon and at a certain confidence level. If the probability distribution of the target criterion is known, the “at risk” measure is expressed as a lower quantile (usually 0.01-quantile or 0.05 quantile) of the distribution in question. It is possible to compute 0.05-quantile out of simulated data, which is in this case ¥ -6.810 mds. and should be seen as the underlying risk measure at confidence 0.95 . It is appropriate to term this quantity as net present value at risk (NPV at risk). With probability 0.95, the project does not attain a lower NPV than ¥ -6.810 mds.
3. Another possibility of constructing “at risk” measures stems from expressing them as an absolute or relative deviation from the expected (or budgeted) level of the target criterion. The value of NPV at risk expressed as an absolute (or relative) deviation captures a maximum negative absolute (or relative) deviation of NPV from the expected amount at a specified confidence level. For the project under assessment, it holds that with confidence 0.95 its NPV will not deviate from the expected value ¥ 3.434 mds. upwards more than 3.434 – (-6.810) = ¥ 10.244 mds., or – put in relative terms – 10.244 / 3.433 = 298.29 %. Both these variants of „at risk“ measures highlight high riskiness of the assessed project.
4. Of heavy importance is the probability with which the project will attain a negative NPV. This figure can easily be estimated as a proportion of negative simulated NPVs to the number of all simulations, which is 33.25 %. In a similar fashion, it is possible to estimate the probability that the NPV of the project will end beneath the expected level. For this project this probability is estimated at 53.63 %. These values are again indicative that this project is highly risky.
5. By comparing risk and return associated with the project, it is possible to estimate acceptability of the project in regard to its risk, or rather to evaluate capability of the project to generate return at a reasonable degree of risk. This may be measured as a proportion of the expected NPV to the standard deviation of NPV. This indicator is useful in comparing favourableness of various projects. For the project in question, a unit of risk (measured by standard deviation) yields expectably 3.434 / 6.946 = 0.49 units of return (in the form of NPV). The relationship between risk and return for this investment project is unbearable and testifies of extreme and inappropriate burdening of the project by risk.

As follows from the aforementioned facts, this project is not recommendable for implementation given the circumstances considered.

5 Conclusion

The paper indicates a possibility of applying the conceptually simple and illustrative simulation method Monte Carlo in investment decisions and project evaluations, which makes it possible to obtain, in a relatively easy way, the probability distribution of NPV and accordingly to determine suitable indicators as inputs to deciding on implementing or discarding the project. A simple case study in the paper demonstrated that this method can be employed so as to obtain answers to questions such as:

1. What is the worst NPV that can be expected for the project at a certain confidence level (usually. 0.95 or 0.99)?
2. What is the probability that the project attains a negative NPV?
3. What is the probability that the NPV of the project is lower than the expected (budgeted) target value?
4. What is the relationship between risk and return for the project?

In spite of unquestionable advantages of this method and despite the fact that Monte Carlo simulations are a very useful tool in deciding on investment projects and in their evaluating, it is needful to recognize that in its final effect it is still a "laboratory" model. No model is capable to map and describe all uncertainties and dependencies, to which a project is subjected. Results obtained through modelling (or Monte Carlo simulations) can only be used as a supportive tool in decision-making and no way can they be globalized or overtopped above results gained through other methods.

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Primary Paper Section: A

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