THE REQUIRED REINFORCEMENT AREA FOR THE CONTROL OF CRACK WIDTHS IN CONCRETE STRUCTURES

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The research described in this paper was developed within and with the support of research project VEGA No. 1/0784/12 “Holistic design and verification of concrete structures”.

Abstract: Visible cracking occurs when the tensile stresses exceed the tensile strength of the material. Visible cracking is frequently a concern since these cracks provide easy access for the infiltration of aggressive solutions into the concrete and reach the reinforcing steel or, other components of the structure leading to deterioration. The design of a structure with reduced width of the cracks can be done using a variety of standards and guidelines. The individual guidelines introduce different procedures for design of the amount of the reinforcement for the cracks width limitation. This paper deals with calculation of the required reinforcement area for the crack widths limitation according to national annexes of the standard EN 1992-1-1, Model Code 2010 and other standards and their differences in the design of the structures.

Keywords: watertight concrete, cracks width, minimum reinforcement area, cracks spacing

1 Introduction

In concrete, mortar and cement paste shrinkage takes place from the very beginning of the life of the material. In early age volume change can be both swelling and shrinking, but later shrinkage is relevant, which is caused by water movement in the porous and rigid body. During the hydration of cement (in the first 2 to 8 hours), while the cement paste is plastic, fresh concrete and cement mortar undergoes a volumetric contraction (plastic shrinkage) and free water content is moving toward the external surface of the specimen. After compaction and subsidence of particles due to its surface tension water is absorbed from the capillary pores towards the external surface and evaporated. Volume reduction of the outer layer is inhibited by the inner part of the material, and this can result map-like wide cracks. During the hydration of cement paste also a volume change occurs (autogenous shrinkage), due to the hydration products (cement matrix) volume is less than the volume of the raw materials (cement + water). However, the extent of hydration prior to setting is small, and once a certain stiffness of the system has developed, the contraction induced by the loss of water due hydration is greatly restrained. Withdrawal of water from concrete in unsaturated air causes drying shrinkage. A part of this drying shrinkage is irreversible and should be distinguished from the reversible moisture movement caused by alternating storage under wet and dry condition. Plastic, autogenous and drying shrinkage together are called early age shrinkage. [3], [6]

Influencing factors of early age shrinkage in mix design:

- cement content of the paste; specific surface area of cement
- fine aggregate content (under 0.125 mm particle size); specific surface area of fine aggregate
- water-cement ratio
- total aggregate content
- type of aggregate; water absorption capacity/water content of aggregate
- applied admixtures
- compacting rate of paste
- porosity
- other added components e.g. fibres.

Shrinkage of concrete depends on the temperature of concrete and its surroundings, on relative humidity and on the velocity of air movement as well as the curing and composition of the concrete. To fulfil the requirements of crack-free structures is often a problem during the design and construction of concrete and reinforced concrete structures, e.g. exposed concretes, hydraulic engineering works, gas- and water- tight concretes. Crack formation is also disadvantageous from the point of view of durability. [3], [6]

2 Design of reinforcement area according to EN 1992-1-1

2.1 The control of cracks width with the direct calculation

This control is based on the conception shown in Fig. 1.

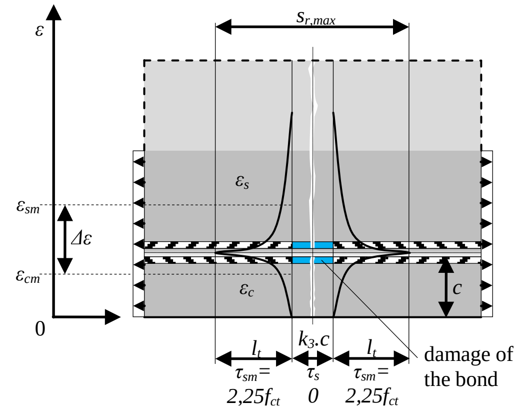

Fig. 1 Conception of calculation of crack width according to STN EN 1992-1-1 [4]

The crack width may be calculated from the expression:

$$w_{s,\max} = s_{r,\max} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \qquad (2.01)$$

where

$s_{r,\max}$ - is the maximum crack spacing [mm]

ε_{sm} - is the mean strain in the reinforcement under the relevant combination of loads

ε_{cm} - is the mean strain in the concrete between cracks

Difference of the mean strains may be calculated from equation:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s}{E_s} - k_t . \frac{f_{ct,eff}}{\rho_{p,eff} . E_s} . (1 + \alpha_e . \rho_{p,eff}) \geq 0.6 \frac{\sigma_s}{E_s} \qquad (2.02)$$

where

σ_s - is the stress in the tension reinforcement assuming a cracked section [kPa]

E_s - design value of modulus of elasticity of reinforcing steel [kPa]

k_t - is a factor dependent on the duration of the load [-]

$f_{ct,eff}$ - is the mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur: $f_{ct,eff} = f_{ctm}$ or lower $(f_{ctm}(t))$, if cracking is expected earlier than 28 days [kPa]

$\rho_{p,ef}$ - is the effective reinforcement ratio [%] $= A_s / A_{c,eff}$

A_s - is the area of reinforcing steel within the tensile zone [m2]

$A_{c,eff}$ - is the effective tension area [m2]

α_e - is the ration E_s / E_{cm} [-]

E_{cm} - is the secant modulus of elasticity of concrete [kPa]

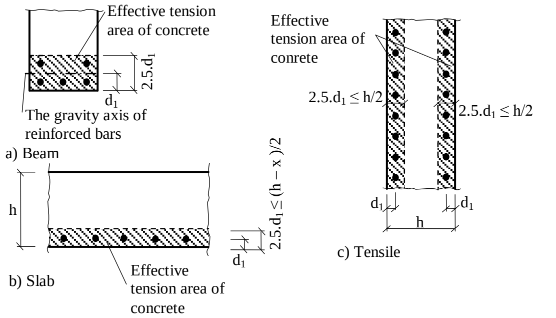

Fig. 2 Effective tension area

The maximum crack spacing calculated from the expression:

$$s_{r,\max} = \begin{cases} k_3.c + k_1.k_2.k_4.\dfrac{d_s}{\rho_{p,eff}} & s \le 5.(c + \dfrac{d_s}{2}) \\ 1.3\cdot(h-x) & s > 5.(c + \dfrac{d_s}{2}) \end{cases} \qquad (2.03)$$

where

c - is the cover to the longitudinal reinforcement [m]

d_s - is the bar diameter [m]

k_1 - is a coefficient which takes account of the bond properties of the bonded reinforcement:
= 0.8 for high bond bars
= 1.6 for bars with an effectively plain surface (e.g. prestressing tendons)

k_2 - is a coefficient which takes account the distribution of strain:
= 0.5 for bending
= 1.0 for pure tension

k_3 - is the recommended value
= 3.4

k_4 - is the recommended value
= 0.425

s - is the spacing of bars [m]

h - is the overall thickness of a cross-section [m]

x - is the neutral axis depth in the stage II [m]

2.1 Minimum reinforcement areas according to EN 1992-1-1

$$A_{s,\min} = k_c.k.f_{ct,eff}.\frac{A_{ct}}{\sigma_s} \qquad (2.04)$$

where

$A_{s,\min}$ - is the minimum area of reinforcing steel within the tensile zone [m²]

k_c - is a coefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm:

For pure tension $k_c = 1.0$

For bending or bending combined with axial forces

- For rectangular section and webs of box sections and T-sections:

$$k_c = 0.4\cdot\left[1 - \frac{\sigma_c}{k_1\cdot\left(h/h^*\right)\cdot f_{ct,eff}}\right] \le 1$$

where

σ_c - is the mean stress of the concrete acting on the part of the section under consideration

h^* — $h^* = h$ for $h < 1.0m$; $h^* = 1.0m$ for $h \ge 1.0m$

k_1 - is a coefficient considering the effects of axial forces on the stress distribution:

$k_1 = 1.5$ if N_{Ed} is a compressive force

$k_1 = \dfrac{2\cdot h^*}{3\cdot h}$ if N_{Ed} is a tensile force

k - is the coefficient which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces
= 1.0 for webs with $h \le 300mm$ or flanges with widths less than 300 mm
= 0.65 for webs with $h \ge 800mm$ or flanges with widths greater than 800 mm
Intermediate values may be interpolated.

A_{ct} - is the area of concrete within tensile zone. The t ensile zone is that part of the section which is calculated to be in tension just before formation of the first crack.

3 Adjustment of calculation according to DIN EN 1992-1-1/NA

DIN EN 1992-1-1/NA introduces the following changes in the calculation:

1.) Changes in the equation for the calculation of the cracks spacing:
   - coefficient $k_3 = 0$, because DIN EN 1992-1-1/NA is not considering the loss of bond of reinforcement and concrete near the crack
   - conjunction of coefficients $k_1 \cdot k_2 = 1$; allowed only high-bond bars
   - coefficient $k_4 = 1/3.6$, which corresponds to the stress in bond $\tau_{sm} = 1.8\cdot f_{ctm}$ for high-bond reinforcement

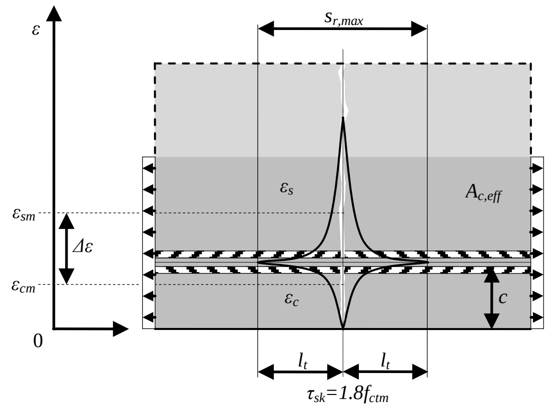

Fig. 3 Conception of calculation of crack width according to DIN EN 1992-1-1/NA [4]

Taking into account the above assumptions of DIN EN 1992-1-1/NA applied for maximum cracks spacing:

$$s_{r,\max} = k_3.c + k_1.k_2.k_4.\frac{d_s}{\rho_{p,eff}} = \frac{d_s}{3.6\cdot\rho_{p,eff}} \le \frac{\sigma_s\cdot d_s}{3.6\cdot f_{ct,eff}} \qquad (3.01)$$

2.) Effective mean value of the tensile strength of the concrete $f_{ct,eff}$ [kPa]
   - if is possible to determine with certainty the formation of cracks in the first 28 days
     $= 0.5\cdot f_{ctm}$ if a crack is created at the time 3 to 5 days
     $= \max\left(f_{ctm};3.0MPa\right)$ - hardening concrete
     $= \max(0.5\cdot f_{ctm};1.5MPa)$ - green concrete
   - if is not possible to determine with certainty the formation of cracks in the first 28 days
     $f_{ctm} \ge 3.0MPa$ - ordinary concrete
     $f_{ctm} \ge 2.5MPa$ - lightweight concrete

3.) Effective tension area $A_{c,eff}$ is determined based on the Fig. 2. However, it is necessary to take into account the extra diagram in Fig. 4, taking into account the effect of the thickness of the element on the effective thickness of the tension area.

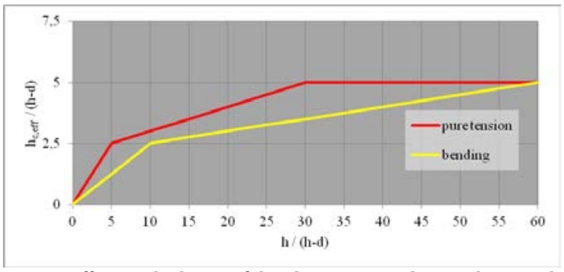

Fig. 4 Effective thickness of the element according to the actual thickness of the element [1]

4.) Changes in the equation for minimum reinforcement area

k - is the coefficient which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces,

- in the presence of tensile stresses induced due the Eigen-stresses from restraints (e.g. a decrease in heat of hydration) 0.8

  k = 0.8 for h ≤ 300mm

  k = 0.5 for h ≥ 800mm

  Intermediate values may be interpolated.

- when the tensile stress is induced by external stress (e.g. slump supports) applies: k = 1.0

4 Calculation according to Model Code 2010

$$w_d = 2.l_{s,\max}.(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}) \tag{4.01}$$

For the length $l_{s,\max}$ the following expression applies:

$$l_{s,\max} = k.c + \frac{1}{4}.\frac{f_{ctm}}{\tau_{bms}}.\frac{\varphi_s}{\rho_{s,ef}} \tag{4.02}$$

where:

k - is an empirical parameter to take the influence of the concrete cover into consideration. As a simplification $k = 1.0$ can be assumed.

c - is the concrete cover

τ_{bm} - is mean bond strength between steel and concrete (Table 1)

The relative mean strain follows from:

$$(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}) = \frac{\sigma_s - \beta.\sigma_{sr}}{E_s} + \eta_r.\varepsilon_{sh} \tag{4.03}$$

where

σ_s - is the steel stress in a crack

σ_{sr} - is the maximum steel stress in a crack in the crack formation stage, which, for pure tension, is:

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{s,ef}}.(1 + \alpha_e.\rho_{s,ef}) \tag{4.04}$$

where

$$\rho_{s,ef} = \frac{A_s}{A_{c,ef}}$$

with $A_{c,ef}$ = effective area of concrete in tension

α_e - is the modular ratio $= E_s / E_{cm}$

β - is an empirical coefficient to assess the mean strain over $l_{s,\max}$ depending on the type of loading (Table 1)

η_r - is a coefficient for considering the shrinkage contribution

ε_{sh} - is the shrinkage strain

Table 1: Values for τ_{bms}, β a η_r for deformed reinforcing bars [5]

	Crack formation stage	Stabilized cracking stage
Short term, instantaneous loading	$\tau_{bms} = 1.8.f_{ctm}(t)$ $\beta = 0.6$ $\eta_r = 0$	$\tau_{bms} = 1.8.f_{ctm}(t)$ $\beta = 0.6$ $\eta_r = 0$
Long term, repeated loading	$\tau_{bms} = 1.35.f_{ctm}(t)$ $\beta = 0.6$ $\eta_r = 0$	$\tau_{bms} = 1.8.f_{ctm}(t)$ $\beta = 0.4$ $\eta_r = 1$

5 Comparisons of standards

The comparisons of standards were made using the program MS Excel. The comparison of the overall required reinforcement area for crack width limitation under all the above standards shows the Fig. 8. Then, the comparisons were made influence of an individual parameters, which focus on differences in the standards STN EN 1992-1-1, DIN EN 1992-1-1/NA, Model Code 2010 (Fig. 5 to Fig. 7). In Fig. 9 is shown the comparison of the eight selected standards.

All comparisons were based on the following assumptions:

- concrete class C25/30 and cement class S,
- reinforcing bars grade B 500B with diameter = 16 mm,
- structural class S3, exposure class XC2, XC3, => the cover to the longitudinal reinforcement c_{nom} = 30 mm,
- maximum crack width $w_{k,max}$ = 0.2 mm,
- age of the concrete t = 5 days.

5.1 Comparison influence of the individual parameters

The approaches of individual standards are nearly identical. The biggest difference between the calculations is the determination of the cracks spacing.

When comparing of the individual parameters to the standards DIN 1992-1-1 and STN EN 1992-1-1 were observed for the slab thickness 2.0 m following the effects:

1. the coefficient k: 16.8% increase of the reinforcement area according to STN EN 1992-1-1,
2. the effective area of the tensile concrete $A_{c,eff}$: 18.8% increase of the reinforcement area according to DIN EN 1992-1-1,
3. the equation for the calculation of the cracks spacing: 35.4% increase of the reinforcement area according to STN EN 1992-1-1.

These differences can be observed for the various slab thicknesses in the following comparisons.

The influence of coefficient *k*

Fig. 5 shows the differences of required reinforcement area due to different values of the coefficient *k*. For comparison, the procedure selected in the required reinforcement according to EN 1992-1-1, the values of the coefficient were taken from the standards DIN EN 1992-1-1/NA and STN EN 1992-1-1

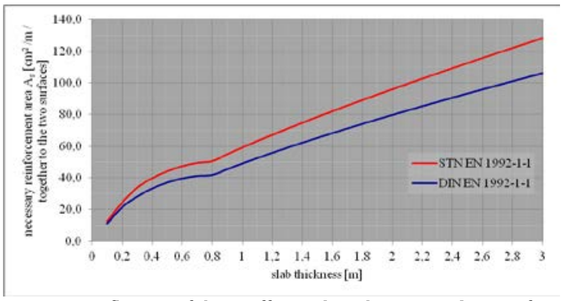

Fig. 5 Influence of the coefficient k to the required area of reinforcement

The influence of the effective area of tensile concrete $A_{c,eff}$

In contrast to other influence has the determination of the effective area on the basis of DIN EN 1992-1-1/NA the opposite effect than any other adjustments that standard, i.e. causes an increase in the required area of the reinforcement. This phenomenon can be observed in Fig. 6. For the calculation of the required area of the reinforcement was used the approach of EN 1992-1-1. The effective area of reinforcement was determined on the basis of DIN EN 1992-1-1/NA and STN EN 1992-1-1/NA.

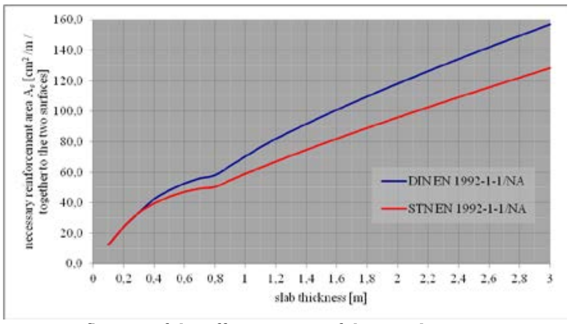

Fig. 6 Influence of the effective area of the tensile concrete $A_{c,eff}$

The influence of the coefficient k_1 to k_4

Given that the equation for calculating the distance between the cracks consists of several coefficients whose values differ depending upon the standard was made compare the influence of coefficients k_1 to k_4 to the required area of reinforcement shows Fig. 7. The values of the coefficients are selected on the basis of the individual standards. The calculation was made according to EN 1992-1-1.

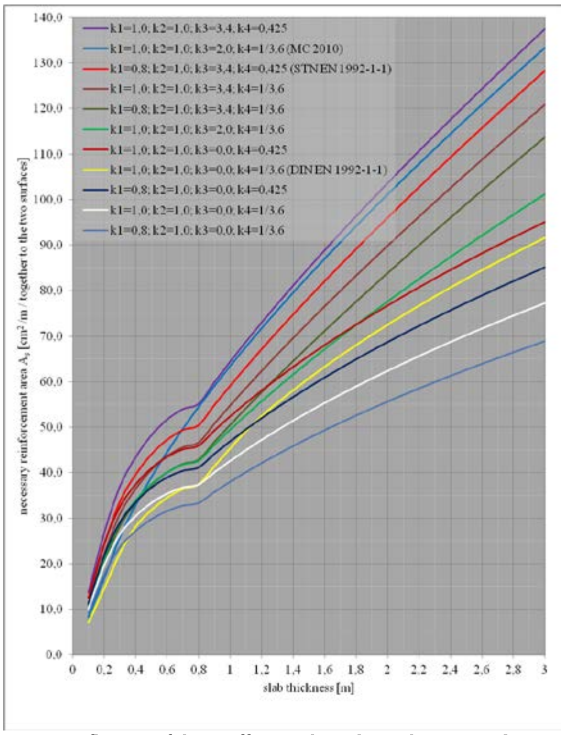

Fig. 7 Influence of the coefficients k_1 to k_4 to the required area of reinforcement

The influence of the equation for the calculation of cracks spacing

Fig. 8 highlights the differences caused by different approaches to calculate the cracks spacing. For comparison, the procedure selected in the draft reinforcement according to EN 1992-1-1, distance calculation cracks was performed using the coefficients k_1 to k_4 according to STN EN 1992-1-1, DIN EN 1992-1-1/NA and the Model Code 2010.

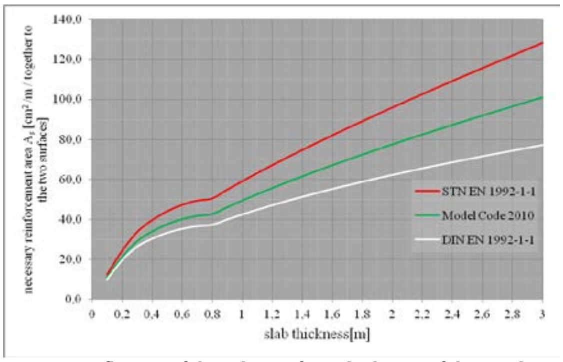

Fig. 8 Influence of the relation for calculating of the cracks spacing on the amount of a required reinforcement

The results of the comparison of the various standards

For the comparison (Fig. 9) of the required reinforcement area for the control of the crack widths were chosen the results of these standards: BS 8007, SIA 262, Model Code 1990, SS EN 1992-1-1, NF EN 1992-1-1, Model Code 2010, STN EN 1992-1-1, DIN EN 1992-1-1.

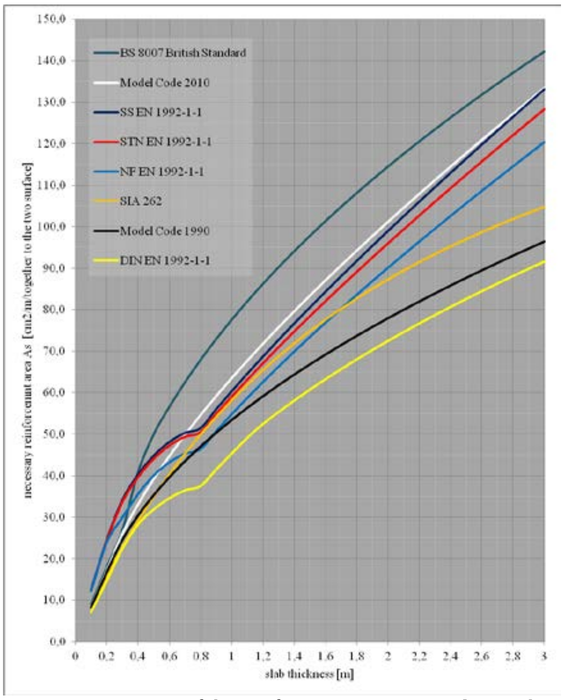

Fig. 9 Comparison of the reinforcement area according to the various standards

6 Conclusions

Cracking in concrete will occur in all but the simplest and smallest of structures. In the structure subjected to hydrostatic pressure a through-crack, of any size, can form a water path, which may result in leakage or wet patches occurring. It is the responsibility of the designer to limit design crack widths to a predetermined size to restrict or prevent water from leaking through the concrete into the basement. The principal and most effective method to control restrained shrinkage and thermal movement cracking is by the provision of sufficient reinforcement. The design approach for early-age thermal cracking adopted by STN EN 1992-1-1:2006 is broadly similar to that of DIN EN 1992-1-1/NA but there are some significant and important differences as follows:

1. the value of the coefficient k, which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces is according to DIN EN 1992-

1-1 lower by the 0.8 times than according to STN EN 1992-1-1,
2. the loss of bond of reinforcement and concrete near the crack in DIN EN 1992-1-1/NA is not taken into the consideration.

The analysis of different standards for the design of reinforcement required to control the crack width revealed, that significant savings in reinforcement area can be obtained using the DIN EN 1992-1-1/NA. Many parameters in STN EN 1992-1-1 vary according to DIN EN 1992-1-1 which causes an increase in steel reinforcement and a significant increase in cost. The main reason for the concentration of our research work to the mentioned topic is to find out the decisive parameters and try to derive their real values. The results demonstrate highly significant differences of the required reinforcement area according to the selected standards.

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Primary Paper Section: J

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