THE REQUIRED REINFORCEMENT AREA FOR THE CONTROL OF Crack WIDTHS IN CONCRETE STRUCTURES

ROBERT SONNENSCHEIN, JURAJ BILCÍK

Department of Concrete Structures and Bridges, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Radlinského 11, 813 68 Bratislava, Slovak Republic. email: +robert.sonnenschein@stuba.sk, juraj.bilcik@stuba.sk

The research described in this paper was developed within and with the support of research project VEGA No. 1/0784/12 “Holistic design and verification of concrete structures”.

In concrete, mortar and cement paste shrinkage takes place from the very beginning of the life of the material. In early age volume change can be both swelling and shrinking, but later shrinkage is relevant, which is caused by water movement in the porous and rigid body. During the hydration of cement (in the first 2 to 8 hours), while the cement paste is plastic, fresh concrete and cement mortar undergoes a volumetric contraction (plastic shrinkage) and free water content is moving toward the external surface of the specimen. After compaction and subsidence of particles due to its surface tension water is absorbed from the capillary pores towards the external surface and evaporated. Volume reduction of the outer layer is inhibited by the inner part of the material, and this can result map-like wide cracks. During the hydration of cement paste also a volume change occurs (autogenous shrinkage), due to the hydration products (cement matrix) volume is less than the volume of the raw materials (cement + water). However, the extent of this drying shrinkage is irreversible and should be distinguished from the reversible moisture movement caused by alternating storage under wet and dry condition. Plastic, autogenous and drying shrinkage together are called early age shrinkage. [3], [6]

Influencing factors of early age shrinkage in mix design:

- cement content of the paste; specific surface area of cement fine aggregate content (under 0.125 mm particle size); specific surface area of fine aggregate
- water-cement ratio
- total aggregate content
- type of aggregate; water absorption capacity/water content of aggregate
- added admixtures
- compacting rate of paste
- porosity
- other added components e.g. fibres.

Shrinkage of concrete depends on the temperature of concrete and its surroundings, on relative humidity and on the velocity of air movement as well as the curing and composition of the concrete. To fulfill the requirements of crack-free structures is often a problem during the design and construction of concrete and reinforced concrete structures, e.g. exposed concretes, hydraulic engineering works, gas- and water- tight concretes. Crack formation is also disadvantageous from the point of view of durability. [3], [6]

2 Design of reinforcement area according to EN 1992-1-1

2.1 The control of cracks width with the direct calculation

This control is based on the conception shown in Fig. 1.

Fig. 1 Conception of calculation of crack width according to STN EN 1992-1-1 [4]

The crack width may be calculated from the expression:

\[ w_{cr,max} = s_{cr,max} \left( \varepsilon_{cm} - \varepsilon_{c} \right) \]  \hspace{1cm} (2.01)

where

- \( s_{cr,max} \) - is the maximum crack spacing [mm]
- \( \varepsilon_{cm} \) - is the mean strain in the concrete under the relevant combination of loads
- \( \varepsilon_{c} \) - is the mean strain in the concrete between cracks

Difference of the mean strains may be calculated from equation:

\[ \varepsilon_{cm} - \varepsilon_{c} = \frac{\sigma_{t}}{E_{c}} \cdot k_{r} \cdot f_{crt,min} \cdot \left( 1 + \rho_{r,eff} \right) \geq 0.06 \cdot \frac{\sigma_{c}}{E_{c}} \]  \hspace{1cm} (2.02)

where

- \( \sigma_{t} \) - is the stress in the tension reinforcement assuming a cracked section [kPa]
- \( E_{c} \) - design value of modulus of elasticity of reinforcing steel [kPa]
- \( k_{r} \) - is a factor dependent on the duration of the load [-]
- \( f_{crt,min} \) - is the mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur: \( f_{crt,min} = f_{ck} \) or lower \( f_{ck} \) if cracking is expected earlier than 28 days [kPa]
- \( \rho_{r,eff} \) - is the effective reinforcement ratio [%]
- \( A_{s} \) - is the area of reinforcing steel within the tensile zone [m²]
- \( A_{eff} \) - is the effective tension area [m²]
- \( \sigma_{c} \) - is the ratio \( E_{c}/E_{cm} \) [-]
- \( E_{cm} \) - is the secant modulus of elasticity of concrete [kPa]
The maximum crack spacing calculated from the expression:

\[ s_{\text{cr},\text{max}} = \frac{k_1 c + k_2 k_3 c + d_s}{\rho_{\text{eff}}} \leq s \leq \frac{5(c + \frac{d_s}{2})}{2} \]

where
- \( c \) - is the cover to the longitudinal reinforcement [m]
- \( d_s \) - is the bar diameter [m]
- \( k_1 \) - is a coefficient which takes account of the bond properties of the bonded reinforcement:
  - 0.8 for high bond bars
  - 1.6 for bars with an effectively plain surface (e.g., prestressing tendons)
- \( k_2 \) - is a coefficient which takes account the distribution of strain:
  - 0.5 for bending
  - 1.0 for pure tension
- \( k_3 \) - is the recommended value
  - 3.4
- \( s \) - is the spacing of bars [m]
- \( h \) - is the overall thickness of a cross-section [m]
- \( x \) - is the neutral axis depth in the section II [m]

2.1 Minimum reinforcement areas according to EN 1992-1-1

\[ A_{\text{c,min}} = k_1 c k_2 f_{\text{ctm}} \frac{A_t}{\sigma_y} \]

where
- \( A_{\text{c,min}} \) - is the minimum area of reinforcing steel within the tensile zone [m²]
- \( k_1 \) - is a coefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm:
  - For pure tension \( k_1 = 1.0 \)
  - For bending or bending combined with axial forces:
    - For rectangular sections and webs of box sections and T-sections:
      \[ k_1 = 0.4 \left[ 1 - \frac{\sigma_y}{k_3 h (b/h)} \right] \leq 1 \]
    - \( \sigma_y \) - is the mean stress of the concrete acting on the part of the section under consideration
    - \( h' = h \) for \( h < 1.0m \)
    - \( h' = 1.0m \) for \( h \geq 1.0m \)
    - \( k_3 \) - is a coefficient considering the effects of axial forces on the stress distribution:
      - \( k_3 = 1.5 \) if \( N_{\text{ax}} \) is a compressive force

3 Adjustment of calculation according to DIN EN 1992-1-1/NA

DIN EN 1992-1-1/NA introduces the following changes in the calculation:

1.) Changes in the equation for the calculation of the cracks spacing:
- coefficient \( k_1 = 0 \), because DIN EN 1992-1-1/NA is not considering the loss of bond of reinforcement and concrete near the crack
- conjunction of coefficients \( k_1 \cdot k_3 \) = 1; allowed only high-bond bars
- coefficient \( k_3 = 1/3 \cdot 6 \), which corresponds to the stress in bond \( \tau_{\text{ax}} = 1.8 \cdot f_{\text{ctm}} \) for high-bond reinforcement

\[ s_{\text{cr},\text{max}} = k_1 c + k_2 k_3 c + \frac{d_s}{\rho_{\text{eff}}} \leq \frac{\sigma_y \cdot d_s}{3.6} \cdot 3.6 \cdot f_{\text{ctm}} \]

Taking into account the above assumptions of DIN EN 1992-1-1/NA applied for maximum cracks spacing:

\[ s_{\text{cr},\text{max}} = k_1 c + k_2 k_3 c + \frac{d_s}{\rho_{\text{eff}}} \leq \frac{\sigma_y \cdot d_s}{3.6} \cdot 3.6 \cdot f_{\text{ctm}} \]

2.) Effective mean value of the tensile strength of the concrete \( f_{\text{ctm}} \) [kPa]

- if is possible to determine with certainty the formation of cracks in the first 28 days:
  - \( f_{\text{ctm}} \) if a crack is created at the time 3 to 5 days
  - \( f_{\text{ctm}} \) for ordinary concrete
  - \( f_{\text{ctm}} \) for lightweight concrete
- if is not possible to determine with certainty the formation of cracks in the first 28 days:
  - \( f_{\text{ctm}} \) for hardening concrete
  - \( f_{\text{ctm}} \) for green concrete

3.) Effective tension area \( A_{\text{ct}} \) is determined based on the Fig. 2. However, it is necessary to take into account the...
extra diagram in Fig. 4, taking into account the effect of the thickness of the element on the effective thickness of the tension area.

![Figure 4: Effective thickness of the element according to the actual thickness of the element [1]](image)

4 Calculation according to Model Code 2010

\[ w_e = 2J_{eff}(e_{cr} - e_{cr} - e_{cr}) \]  \hspace{2cm} (4.01)

For the length \( l_{cr} \) the following expression applies:

\[ l_{cr} = k_c + \frac{1}{4} \tau_{cr} \rho_{cr} \]  \hspace{2cm} (4.02)

where:
- \( k \) - is a coefficient which allows for the effect of the thickness of the element on the effective thickness of the tension area
- \( c \) - is the coefficient which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces,
- \( \tau_{cr} \) - is mean bond strength between steel and concrete
- \( \rho_{cr} \) - is the steel stress in a crack

The relative mean strain follows from:

\[ (e_{cr} - e_{cr} - e_{cr}) = \frac{\sigma - \beta \sigma}{E_e} + \eta \varepsilon_{cr} \]  \hspace{2cm} (4.03)

where:
- \( \sigma_e \) - is the steel stress in a crack
- \( \sigma_{cr} \) - is the maximum steel stress in a crack in the crack formation stage, which, for pure tension, is:

\[ \sigma_{cr} = \frac{\tau_{cr}}{\rho_{cr}(1 + \alpha_{cr} \rho_{cr})} \]  \hspace{2cm} (4.04)

where

\[ \rho_{cr} = \frac{A_{cr}}{A_{def}} \]

with \( A_{def} \) = effective area of concrete in tension

\[ \alpha_{cr} \] - is the modular ratio \( E_e / E_{cr} \)

\[ \beta \] - is an empirical coefficient to assess the mean strain over \( l_{cr} \), depending on the type of loading (Table 1)

\[ \eta \] - is a coefficient for considering the shrinkage contribution

\[ \varepsilon_{cr} \] - is the shrinkage strain

5 Comparisons of standards

The comparisons of standards were made using the program MS Excel. The comparison of the overall required reinforcement area for crack width limitation under all the above standards shows the Fig. 8. Then, the comparisons were made influence of an individual parameters, which focus on differences in the standards STN EN 1992-1-1, DIN EN 1992-1-1/NA, Model Code 2010 (Fig. 5 to Fig. 7). In Fig. 9 is shown the comparison of the eight selected standards.

All comparisons were based on the following assumptions:
- concrete class C25/30 and cement class S
- reinforcing bars grade B 500B with diameter \( = 16 \) mm
- structural class S3, exposure class XC2, XC3
- the cover to the longitudinal reinforcement \( c_{nom} = 30 \) mm
- maximum crack width \( w_{cr,max} = 0.2 \) mm
- age of the concrete \( t = 5 \) days

5.1 Comparison influence of the individual parameters

The approaches of individual standards are nearly identical. The biggest difference between the calculations is the determination of the cracks spacing.

When comparing of the individual parameters to the standards DIN 1992-1-1 and STN EN 1992-1-1 were observed for the slab thickness 2.0 m following the effects:

1. the coefficient \( k \): 16.8% increase of the reinforcement area according to STN EN 1992-1-1,
2. the effective area of the tensile concrete \( A_{def} \): 18.8% increase of the reinforcement area according to DIN EN 1992-1-1,
3. the equation for the calculation of the cracks spacing: 35.4% increase of the reinforcement area according to STN EN 1992-1-1.

These differences can be observed for the various slab thicknesses in the following comparisons.

The influence of coefficient \( k \)

Fig. 5 shows the differences of required reinforcement area due to different values of the coefficient \( k \). For comparison, the procedure selected in the required reinforcement according to EN 1992-1-1, the values of the coefficient were taken from the standards DIN EN 1992-1-1/NA and STN EN 1992-1-1

![Figure 5: Influence of the coefficient \( k \) to the required area of reinforcement](image)
The influence of the effective area of tensile concrete $A_{\text{eff}}$

In contrast to other influences, there is the determination of the effective area of concrete on the basis of DIN EN 1992-1-1/NA. The opposite effect than any other adjustments that standard, i.e. causes an increase in the required area of reinforcement. This phenomenon can be observed in Fig. 6. For the calculation of the required area of reinforcement was used the approach of EN 1992-1-1. The effective area of reinforcement was determined on the basis of DIN EN 1992-1-1/NA and STN EN 1992-1-1/NA.

![Fig. 6 Influence of the effective area of the tensile concrete $A_{\text{eff}}$.](image)

The influence of the coefficient $k_1$ to $k_4$

Given that the equation for calculating the distance between the cracks consists of several coefficients whose values differ depending upon the standard was made compare the influence of coefficients $k_1$ to $k_4$ to the required area of reinforcement shows Fig. 7. The values of the coefficients are selected on the basis of the individual standards. The calculation was made according to EN 1992-1-1.

![Fig. 7 Influence of the coefficients $k_1$ to $k_4$ to the required area of reinforcement.](image)

The influence of the equation for the calculation of cracks spacing

Fig. 8 highlights the differences caused by different approaches to calculate the cracks spacing. For comparison, the procedure selected in the draft reinforcement according to EN 1992-1-1, distance calculation cracks was performed using the coefficients $k_1$ to $k_4$ according to STN EN 1992-1-1, DIN EN 1992-1-1/NA and the Model Code 2010.

![Fig. 8 Influence of the relation for calculating of the cracks spacing on the amount of a required reinforcement.](image)

The results of the comparison of the various standards

For the comparison (Fig. 9) of the required reinforcement area for the control of the crack widths were chosen the results of these standards: BS 8007, SIA 262, Model Code 1990, SS EN 1992-1-1, NF EN 1992-1-1, Model Code 2010, STN EN 1992-1-1, DIN EN 1992-1-1.

![Fig. 9 Comparison of the reinforcement area according to the various standards.](image)

6 Conclusions

Cracking in concrete will occur in all but the simplest and smallest of structures. In the structure subjected to hydrostatic pressure a through-crack, of any size, can form a water path, which may result in leakage or wet patches occurring. It is the responsibility of the designer to limit design crack widths to a predetermined size to restrict or prevent water from leaking through the concrete into the basement. The principal and most effective method to control restrained shrinkage and thermal movement cracking is by the provision of sufficient reinforcement. The design approach for early-age thermal cracking adopted by STN EN 1992-1-1:2006 is broadly similar to that of DIN EN 1992-1-1/NA but there are some significant and important differences as follows:

1. the value of the coefficient $k$, which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces is according to DIN EN 1992-
1. lower by the 0.8 times than according to STN EN 1992-1-1,
2. the loss of bond of reinforcement and concrete near the crack in DIN EN 1992-1-1/NA is not taken into the consideration.

The analysis of different standards for the design of reinforcement required to control the crack width revealed, that significant savings in reinforcement area can be obtained using the DIN EN 1992-1-1/NA. Many parameters in STN EN 1992-1-1 vary according to DIN EN 1992-1-1 which causes an increase in steel reinforcement and a significant increase in cost. The main reason for the concentration of our research work to the mentioned topic is to find out the decisive parameters and try to derive their real values. The results demonstrate highly significant differences of the required reinforcement area according to the selected standards.

Literature:

Primary Paper Section: J

Secondary Paper Section: JM, JN