ASSESS THE ABILITY TO INTERCEPT CRUISE MISSILES OF SURFACE-TO-AIR MISSILE SYSTEM

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Abstract: With the continuous development of technology, missile systems, especially cruise missiles (CM) have become an extremely dangerous threat to national security. The CM can pass through the defense zone without being detected, they are equipped with systems used for electronic war and are capable of flying hundreds of kilometers. CMs are small in size, so they are easily stored and transported by many different methods. For these reasons, building and evaluating the ability of the CM defense system is essential. The paper will investigate the effects of various factors: giudance laws and noise to the ability to intercept CM by the surface-to-air missile defense system (SAM). The guidance laws investigated in the paper are the proportional navigation (PN) guidance law and differential geometry (DG) guidance law. The simulations on MATLAB software will show the ability to intercept CM in different situations.

Keywords: Missile system, Cruise missile, Surface-to-air missile, Guidance law, Proportional navigation guidance law.

1 Introduction

The cruise missile has demonstrated its ability to damage from the beginning of World War II when Germany launched more than 200 cruise missiles towards England and Belgium. (1) Although at that time, cruise missiles still lacked of accuracy but they caused thousands of deaths. More recently, during the Iraq War in 2003, Kuwaiti and US ballistic missile defense systems failed to detect and intercept 5 Seersucker HY-2/CSSC-3 cruise missiles of Iraq. (1)

The CM can fly at subsonic speeds, can also fly at supersonic speeds (up to 3 Mach), at the distance up to 3000 km. (2) Modern CM missiles can pass through missile and radar systems without much difficulty. Using high-precision guidance systems such as GPS (global positioning system) or GLONASS allows the CM to fly thousands of kilometers to destroy the target with a deviation within a few meters. Another reason that makes CM become a dangerous target for CM missile defense systems is that it can be easily transported, it is difficult to detect because it is small in size and mobile.

With the continuous development of CM, the construction, consolidation, and strengthening of the missile defense system to cope with different types of target, especially cruise missiles is always essential to ensure national security. One of the most important components of a missile is the guidance law, which determines the necessary acceleration command for a missile to fly in the desired orbit. (3) For that reason, it is necessary to investigate and evaluate the ability to intercept cruise missiles, whereby we can make changes to enhance the ability to intercept cruise missiles of the missile defense system.

2 Purpose

The paper will investigate the effect of guidance laws and noise on the ability to intercept CM by the surface-to- air missile defense system, thereby synthesizing the most optimal guidance law.

3 Research Method

The paper will use the three-degree-of-freedom model (3DOF) to model the dynamics of missiles. Specifically, the defense missile used for the simulation will be based on the technical parameters of PAC-3 missile (4), while the target model (cruise missile) will be based on the technical parameters of Tomahawk cruise missile. (5)

In simulations, missiles are modeled in NED coordinate system (North - East - Down coordinate system). The NED coordinate system has the origin located on the tangent plane of the Earth. The X axis of the coordinate system points to the north, the Y axis points to the east. To match the right hand rule, the Z axis of the coordinate system will point vertically to the center of the Earth (3) (Figure 1). In figures 1, η_M and η_T are respectively the guidance angles of the missiles and targets; $\vec{V}_M$ and $\vec{V}_T$ are respectively the velocity vectors of missiles and targets; $\theta_{L_{az}}$ and $\theta_{L_{el}}$ are respectively the line-of-sight (LOS) angles for the azimuth and deviant. Assume that the earth is flat and ignores the effect of angular velocity in the rotation of the earth.

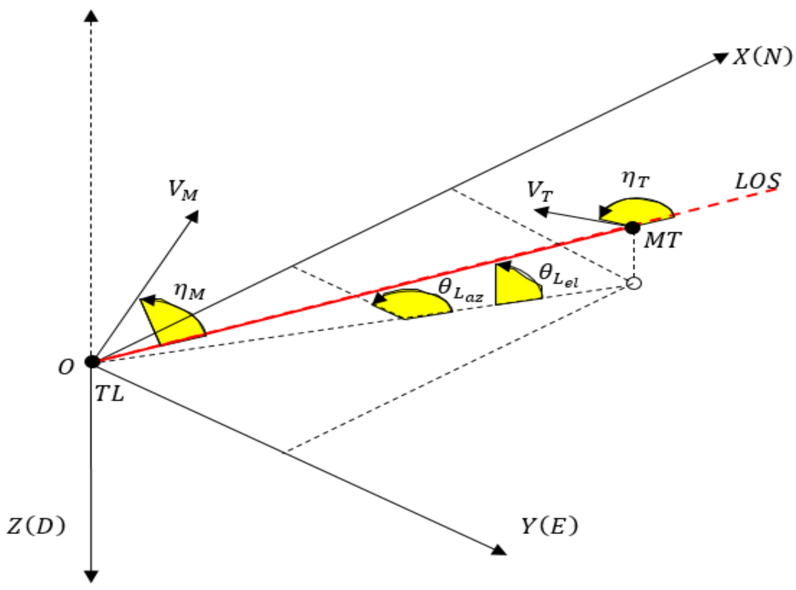

Figure 1. Missiles and Targets in the NED Coordinate System

The vectors or matrices associated with the missile will be added to the index under M; while the parameters related to the target will be added index under T.

3.1 Missile Models

In simulations, missiles will be modeled according to the specifications of PAC-3 missile. Because the maneuverability of missile is always limited due to physical factors, it is necessary to obtain a linear accelerator limiter. In simulations, the paper will limit the guidance acceleration to no more than 50g. The technical parameters of PCA-3 missile are given in Table 1 below (4):

Table 1. The Technical Parameters of PCA-3 Missile, which are used for Missile Models

Length	5.205 m
Diameter	255 mm
Mass	Launch: 315 kg Collision: 142 kg
Speed	1700 m/s
Distance	15 km
Warhead	Kinetic energy
Guidance	INS, active radar
Type of propellant	Solid fuel (175 kg of HTPB)

a) Movement of missiles
At the initial time, the missile position coincides with the origin. Assume that there is no deviation of initial guidance angle. Before launching, missiles are headed straight to the target. The target starts on the positive part of the X axis and is at a distance away from the missile equal to the investigated distance (from 2 km to 15 km). The initial guidance angle of the target is set to the desired deviation angle. The paper investigates with surface-to-air missiles and they are launched from the ground, so it is necessary to define the parameters of vertical lauching angle, which will be set from 15^o to 41^o and calculated according to the following expression:

$$VLA = 15 + 2(D_T - 2) \qquad (1)$$

In which, D_T (km) is the initial distance to the target. The missile status vectors with velocity components and distances along X, Y and Z axes are used to calculate missile motion from time to time. The missile status vector $M(k)$ at the k^{th} time has the form of:

$$M(k) = [x_M(k)\dot{x}_M(k)y_M(k)\dot{y}_M(k)z_M(k)\dot{z}_M(k)]' \qquad (2)$$

and the missile status vector at the next time is $M(k+1)$ which is determined from the transition matrix and missile acceleration matrix as follows:

$$M(k+1) = F_M M(k) + M_a(k) = \cdots$$

$$\cdots \begin{bmatrix} 1 & \Delta & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_M(k) \\ \dot{x}_M(k) \\ y_M(k) \\ \dot{y}_M(k) \\ z_M(k) \\ \dot{z}_M(k) \end{bmatrix} + \begin{bmatrix} \frac{\ddot{x}_M(k)\Delta^2}{2} \\ \ddot{x}_M(k)\Delta^2 \\ \frac{\ddot{y}_M(k)\Delta^2}{2} \\ \ddot{y}_M(k)\Delta^2 \\ \frac{\ddot{z}_M(k)\Delta^2}{2} \\ \ddot{z}_M(k)\Delta^2 \end{bmatrix} \qquad (3)$$

b) Missile thrust
A thrust model which is close to reality requires the following parameters:
Combustion time, thrust /weight ratio (T/W) and specific impulse (I_{sp}). These parameters are subjectively determined because there are no open documents that provide accurate information about these parameters. The T/W ratio of the MIM-104A PAC-2 missile is 15.57 to be used for the PAC-3 missile. (6) The missile which has a single-stage propulsion with solid fuel that is hydroxyl-terminated polybutadiene (HTPB). The specific impulse value I_{sp} of this fuel engine is between 260 s and 265 s. (7)

The paper will choose I_{sp} = 260 s. With the assumption of T/W ratio and I_{sp} above, missile thrust T và burn time t_{burn} calculated as follows:

$$T = (T/W).mass_{total}.g \qquad (4)$$

$$t_{burn} = \frac{I_{sp}(mass_{total} - mass_{impact})g}{T} \qquad (5)$$

In which, $mass_{total}$ is the mass of missiles at launch time; $mass_{impact}$ is the mass of missiles after the fuel has burned out; g is the earth gravitational acceleration. At that time, in the acceleration phase of the flight process, the calculated values of T and t_{burn} are respectively 48.087 (N) and 9.1711 (s)
Assume that the speed of air discharge and fuel flow is constant to ensure constant thrust. At that time, missile acceleration will increase over time because missile mass will decrease over time due to fuel combustion process. (3)

c) Resistance model
The paper only considers two main types of resistance that affect the flight process of missiles are: inductive resistance and parasitic resistance. A missile with a mass of m at a specific time, the total resistance is equal to the total of two component resistance. The total resistance vector has a direction opposite to the direction of the missile velocity vector. The magnitude of the acceleration caused by the resistance is calculated as follows:

$$\left\| a_{M_{drag}} \right\| = \frac{F_{dp} + F_{dl}}{m} \qquad (6)$$

In which, F_{dp} is the parasitic resistance, F_{dl} is the inductive resistance.
The parasitic resistance F_{dp} is proportional to the parasitic resistance coefficient C_{dp} and the horizontal cross-section area S_{REF} according to the following expression (8):

$$F_{dp} = QC_{dp}S_{REF} \qquad (7)$$

Q is dynamic pressure, depending on missile speed v_M and flight altitude according to the following expression (8):

$$Q = \frac{\rho \|v_M\|^2}{2} \qquad (8)$$

For inductive resistance, it can be determined through parasitic resistance, the magnitude of the linear acceleration command and the value of maximum allowed acceleration command of the missile as follows (9):

$$F_{di} = 4F_{dp}\frac{\|a_c\|}{a_{cmax}} \quad (9)$$

3.2 Target Model

In simulations, the target is modeled according to the parameters of the Tomahawk cruise missile. The target is modeled as a material point in the NED coordinate system with the origin coinciding with the missile launch point. The initial position of the target is set on the positive side of X axis of the coordinate system. With 3DOF simulations, the target will maintain the flight speed and altitude is respectively 249.6312 m/ s and 276.7584 m. (5)

The target maintains a linear trajectory before the encounter time of 3 seconds. The effect of resistance and gravity on the target will not be considered in the paper. In the last 3 seconds before the end of the flight process, the target (cruise missile) performs maneuvering of 6.89 g and that is also the maximum overload value of the Tomahawk missile. (5)

Using concepts similar to those in the part of building a missile model, we obtain the target state vector at the time k is $T(k)$ as follows:

$$T(k) = [x_T(k)\dot{x}_T(k)y_T(k)\dot{y}_T(k)z_T(k)\dot{z}_T(k)]' \quad (10)$$

When the target does not perform any initial maneuver, the target's linear trajectory is determined based on the target transition state matrix F_T as follows:

$$T(k+1) = F_T T(k) = \begin{bmatrix} 1 & \Delta & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_T(k) \\ \dot{x}_T(k) \\ y_T(k) \\ \dot{y}_T(k) \\ z_T(k) \\ \dot{z}_T(k) \end{bmatrix} \quad (11)$$

When the target performs maneuvering, we can represent the target state vector at the time of $k+1$ according to the target transition matrix F_{turn} as follows (10):

$$T(k+1) = F_{turn}T(k) = \cdots$$

$$= \cdots \begin{bmatrix} 1 & \sin\frac{\omega_{turn}\Delta}{\omega_{turn}} & 0 & \frac{1-\cos(\omega_{turn}\Delta)}{\omega_{turn}} & 0 & 0 \\ 0 & \cos(\omega_{turn}\Delta) & 0 & -\sin(\omega_{turn}\Delta) & 0 & 0 \\ 0 & \frac{1-\cos(\omega_{turn}\Delta)}{\omega_{turn}} & 1 & \sin\frac{\omega_{turn}\Delta}{\omega_{turn}} & 0 & 0 \\ 0 & \sin(\omega_{turn}\Delta) & 0 & \cos(\omega_{turn}\Delta) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_T(k) \\ \dot{x}_T(k) \\ y_T(k) \\ \dot{y}_T(k) \\ z_T(k) \\ \dot{z}_T(k) \end{bmatrix} \quad (12)$$

3.3 Noise Model

The missile model used in the paper is working with active radar systems on the compartment and it is used to measure the parameters of LOS angle and distance, those measurements are affected by noise. The noise model will be added the sensor deviation so the measurement is more close to the reality.

Noise is added to the distance, azimuth LOS angle and LOS angle according to the following expressions:

$$\theta^*_{L_{az}} = \theta_{L_{az}} + \sigma_{\theta_L} n_{rand} \quad (13)$$

$$\theta^*_{L_{el}} = \theta_{L_{el}} + \sigma_{\theta_L} n_{rand} \quad (14)$$

$$r^* = r + \sigma_r n_{rand} \quad (15)$$

Expressions (13, 14, 15) represent measurements that have been affected by noise; σ_{θ_L} and σ_r are respectively the standard deviations of LOS angle and distance measurements by noise. The n_{rand} parameter is the Gauss random variables with a variance of 1, which simulates the noise that affects the measurements. The variance of the measurements will be determined by multiplying the base standard deviation by the noise factor f_{noise}:

$$\sigma_{\theta_{Laz}} = f_{noise}\sigma_{L(base)} \quad (16)$$

$$\sigma_{\theta_{Lel}} = f_{noise}\sigma_{L(base)} \quad (17)$$

$$\sigma_r = f_{noise}\sigma_{r(base)} \quad (18)$$

In which, f_{noise} is a multiplier factor that is used in general to synchronously increase the influence of noise on measurements.

With 3DOF model, conductive base standard deviation for the distance $\sigma_{r(base)}$ is 10.0 m; conductive base standard deviation for deviant and azimuth angles $\sigma_{\theta_{L(base)}}$ is 1.0 mrad.

3.4 PN and DG Guidance Laws

a) PN guidance laws

PN guidance laws have 2 versions: TPN and PPN. In keeping with the reality, the paper investigates with PPN giudance laws. At that time, the acceleration command vector is perpendicular to the missile velocity vector. (11) With the assumption of a small attack angle, we obtain the acceleration command vector which is perpendicular to the vertical axis of the missile. The expression that defines the missile acceleration command is shown below (11):

$$a_c = \frac{N' v_c \dot{\theta}_L}{\cos\eta_M} \quad (19)$$

In which, η_M is the angle between the missile velocity vector and LOS vector, also known as the missile guidance angle (Figure 1).

b) DG guidance laws

The concept of differential geometry has been used as a basis for developing more generalized guidance methods. In the document (12), the author has given the expression to determine the linear acceleration command which is perpendicular to the missile velocity as follows:

$$a_c = \|a_T\|\frac{\cos\eta_T}{\cos\eta_M} + \frac{N' v_c \dot{\theta}}{\cos\eta_M} \quad (20)$$

The guidance angles of the target and missile are symbolized as η_T and η_M. These angles are shown in Figure 1. The magnitude of the target acceleration, $\|a_T\|$, and the target guidance angle, η_T, carrying the information about the target trajectory curvature.

4 Investigating Method

Simulations will be performed at specific distances and angles. The investigating angle will be gradually increased from 0° to 180°. The simulation will stop when:

- The sign of the approaching velocity starts to change from positive to negative.
- When the missile meets the target or when the distance is less than 5m.
- The height is less than 0.
- In addition, there will be constraints approaching velocity to
- Avoid the phenomenon that the simulation stops before applying the guidance laws

- Ensure the missile velocity is greater than the target velocity.

The quality of the guidance laws will be assessed through three parameters: the encounter time, the encounter velocity and the integral of the linear acceleration command of the flight process, this is an important parameter because it allows to obtain a relative comparision of the amount of fuel consumed. For the parameter of encounter time, we always want this parameter to be as small as possible. Since the paper investigates with the presence of noise, at each investigating angle value, the result will be the average value of the parameters after 20 simulations. The paper will carry out PN and DG guidance laws at 3 values of distance of 2 km, 8 km and 15 km. The investigating results in the above 3 values of distance will be shown on one graph to facilitate the comparison.

5 Results

The results at 3 different investigating distances: 2 km, 8 km, and 15 km will be performed on one graph at the investigating angle from 0o to 180o to facilitate the comparison.

When we investigate at the farthest distance that is 15 km, the missile misses the target when the investigating angle is less than 20o, so the investigating results at a distance of 15 km with the investigating angle range from 0o to 20o will not be shown on the diagram showing the difference between the parameters when applying PN and DG guidance laws (Figure 2a, Figure 3a and Figure 4a). To clarify the investigating results at a distance of 15 km, the parameters evaluating the effectiveness of PN and DG guidance laws will be presented on separate graphs (Figure 2b, Figure 3b and figure 4b).

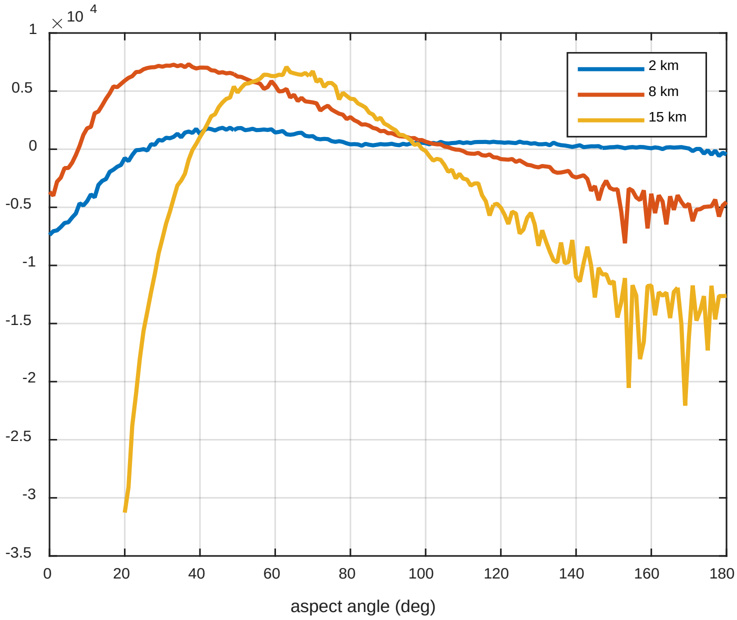

Figure 2a. The Difference between the Integrals of the Magnitude of Linear Acceleration Command of PN and DG Guidance Laws

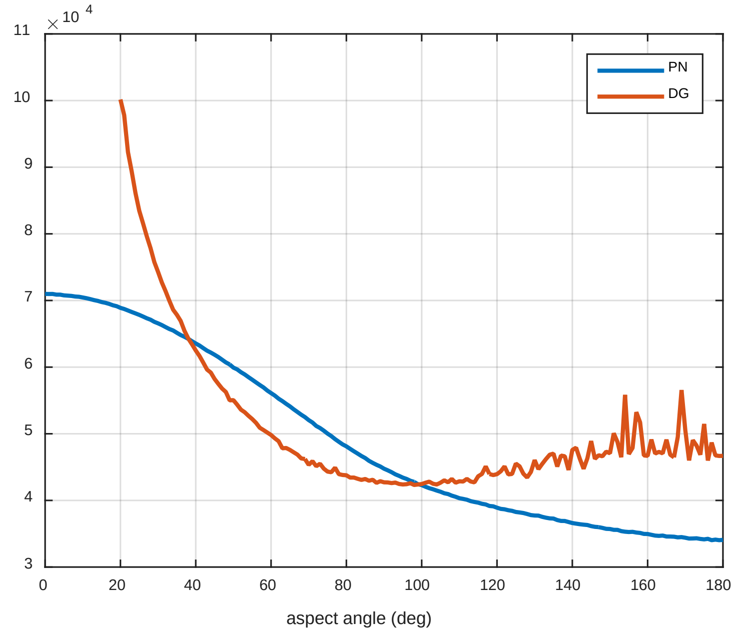

Figure 2b. The Integral of the Magnitude of Linear Acceleration Command of PN and DG Guidance Laws When Investigating at a Distance of 15 km

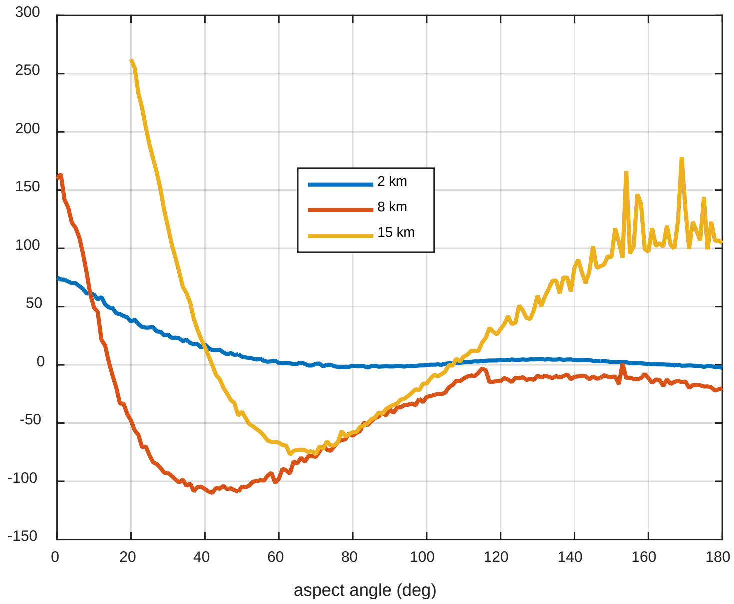

Figure 3a. The Difference between the Missile Collision Velocities When Applying PN and DG Guidance Laws at 3 Distances

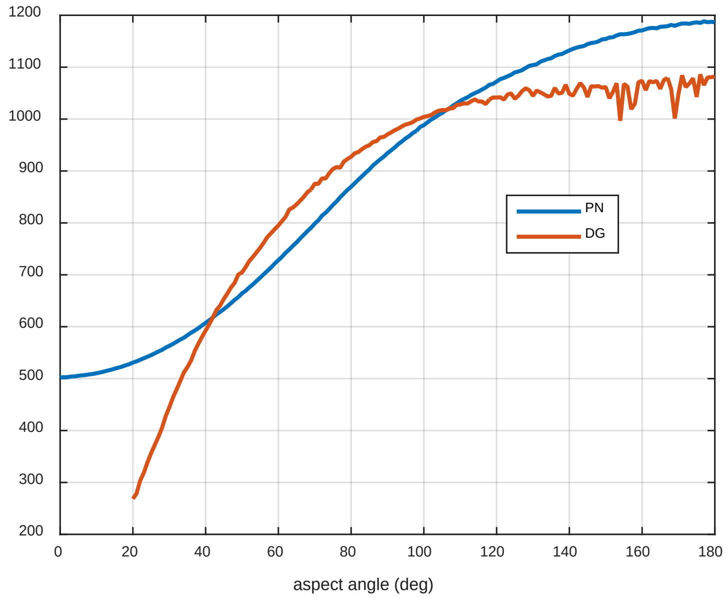

Figure 3b. The Missile Collision Velocities When Applying PN and DG Guidance Laws at a Distance of 15 km

From Figure 2a we can see that the difference between the two guidance laws is more obvious when increasing the investigating distance. At the investigating distance of 2 km, the DG guidance law appears to be dominant compared to the PN guidance law when the integral of the magnitude of the guidance command is smaller than that in the case of the PN guidance law in most investigating angles. When the investigating distance is increased to 8 km and 15 km, the investigating angle that DG guidance law governed was narrowed. On the whole investigating distance from 2 km to 15 km, we can say that the integration parameters of the magnitude of the guidance command when applying DG guidance law are always better (smaller) than those in case of applying the PN guidance law in the investigating angle range from 40o to 97o (in case of chasing - target flying out).

Figure 3a shows the difference between the missile collision velocities when applying PN guidance law and when applying DG guidance law, this is perfectly consistent with Figure 2a. That is, the missile velocity will decrease due to the resistance and gravity when the missile performs maneuvering, so the more the missile performs maneuvering, the more the missile's collision velocity will decrease.

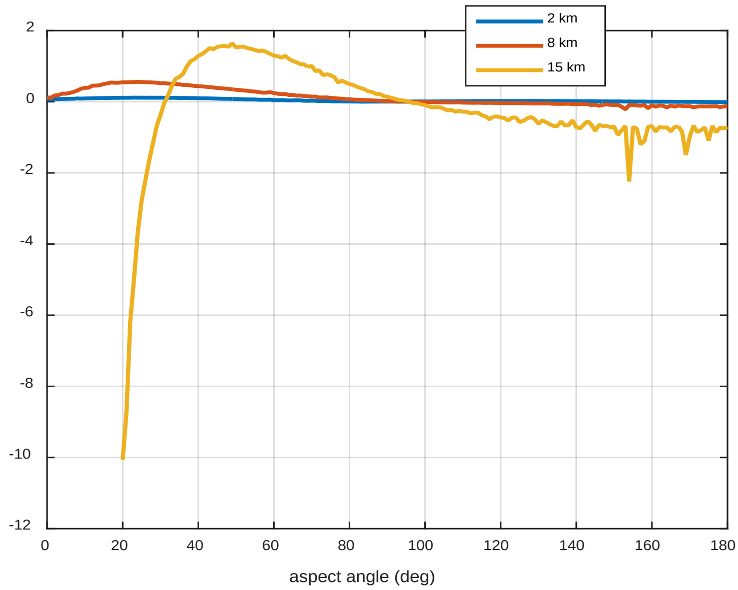

Figure 4a. The Difference Between the Collision Time When Applying PN and DG Guidance Laws at 3 Investigating Distances

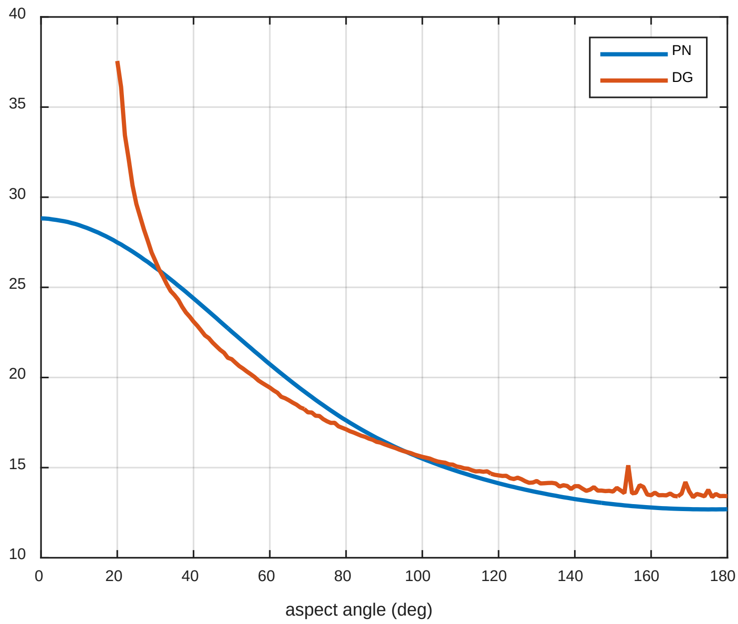

Figure 4b. The Collision Time When Applying PN and DG Guidance Laws at a Distance of 15 km

Figure 4a compares the missile collision time when applying the two guidance laws. Similar to the previous two parameters, the DG guidance law appears to be dominant in the case of chasing the target on the investigating angle range from 31^o to 97^o.

6 Scientific and Practical Significance

The paper used the three-degree-of-freedom model to simulate the situation of intercepting the cruise target (modeling according to the parameters of the Tomahawk missile) of surface-to-air missiles (modeling according to the parameters of the PAC-3 missile). From the obtained parameters of quality evaluation of the guidance laws from the simulation, we can see that when shooting the flight target at small distance with a constant velocity, both PN and DG guidance laws have its own limitations. The obtained investigating results have a practical significance in the process of improving the effectiveness of intercepting the cruise target of air defense missile system. The ability to destroy the target is improved while the time required to destroy the target is shortened when building a guidance system that allows to choose between PN and DG guidance laws based on the angle position of the target.

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