ON THE SOLUTION OF ONE MODIFIED ASSIGNMENT PROBLEM

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Abstract: This article sets out and provides an algorithm for solving one modified assignment problem. The task of selecting a team of project executors is considered, the distribution of private tasks of the project among potential executors is carried out. When solving the problem, the assessments of the compliance of the performers with the requirements of individual tasks, the complexity of the tasks, the degree of employment of the performers in other projects, the distribution of the sequence of tasks by the deadlines are taken into account. The article proposes an algorithm for obtaining assessments of the compliance of the contractor with the requirements of a particular task based on a set of characteristics of the contractor, the key characteristics are assessments of professional competence. Assessment of compliance with the requirements of tasks is carried out on the basis of expert assessments using fuzzy relationship preferences. To obtain estimates of the value of characteristics, experts construct matrices of a fuzzy relationship of preference on the set of all characteristics. An objective function is constructed that allows one to obtain an estimate of each possible distribution. To take into account the restrictions imposed on possible distributions, the corresponding terms in the form of penalty functions are added to the objective function, which significantly reduce the distribution estimate in case of failure to fulfill the specified restrictions. To solve the formulated problem, which is actually a combinatorial optimization problem, a standard genetic algorithm is used.

Keywords: project, genetic algorithm, selection of performers, fuzzy estimates, combinatorial optimization, assignment problem.

1 Introduction

The search for effective algorithms for solving human resources management tasks does not lose its relevance even now, which is due to both an increase in the requirements for professional qualifications of employees and an increase in the complexity of professional tasks to be solved. Each new project in any field of activity will be successful if they meet the requirements for the tasks being implemented, the level of professional competence, experience and psychological and personal characteristics of the project executors, that is if individual tasks of the project are correctly distributed among potential executors. This task is an optimization problem. In the case of a small number of options for the possible distribution of tasks, it is solved by exhaustive search, and in the case of a large number of options, combinatorial optimization methods are used. Similar tasks in optimization theory are called assignment problems. For the assignment problem, there are exact solution methods, for example, the branch and bound method and dynamic programming. However, in cases where modified models of the assignment problem are used, known methods often become inapplicable (Zakharova & Minashina, 2015; Greshilov, 2006; Ovchinnikov, 2015; Medvedeva, 2013; Asanov & Myshkina, 2017). For this reason, approximate methods, for example, neural networks and genetic algorithms, are often used to solve such problems (Hung & Wang, 2003; Jin et al., 2003; Siqueira et al., 2007).

In this paper, we consider a possible approach to solving the modified assignment problem, in which there is a need to take into account the complexity of the private tasks of the project. To search for the extreme value of the objective function, a standard genetic algorithm is used, which is often used to solve combinatorial optimization problems.

2 Statement Of The Problem

A calendar plan has been drawn up for the implementation of private tasks of the project $\{T_i\}$ ($i=\overline{1,m}$, m - the total number of tasks), each stage includes many tasks $\{F(s)\}$ ($s=\overline{1,K}$, $\{F(s)\}\subset\{T_i\}$, K - the total number of stages). There are known estimates of the degree of employment of performers in other projects $\{z_i\}$ ($i=\overline{1,n}$, $z_i\in[0,1]$, n - the total number of potential performers). The sequence of tasks and the complexity of each task $\{s_i\}$ ($i=\overline{1,m}$) are known. Based on the estimates of the complexity of each task, the number of performers $\{p_i\}$ ($i=\overline{1,m}$) who will perform this task can be determined, we will consider it known. It is necessary to form a team of performers and distribute m tasks between them in such a way that the predicted assessment of the success of the project is maximum under the conditions that: (1) each task performs the specified number of performers; (2) the contractor can perform several tasks at different stages of the project and at one stage; (3) the total complexity of the tasks performed by one performer at one stage does not exceed the specified value S.

3 Algorithm For Assessing The Compliance Of Potential Performers With The Requirements Of A Private Project Assignment

To predict the success of the project and the formation of the objective function, allowing to assess the possible distribution of tasks, it is necessary to assess the compliance of potential performers with the requirements of individual tasks of the project. To obtain these estimates, the approaches used for evaluating and selecting job seekers are applicable (Asanov, 2015; Asanov & Myshkina, 2010). We believe that the individual assessments of each artist for each criterion are known and take values from 0 to 1; for estimates taking values from a range other than the interval [0; 1], in this case, the normalization operation can be applied.

This algorithm uses the apparatus of the theory of fuzzy sets and fuzzy logic (Asanov & Myshkina, 2010; Piegat, 2013).

Definition of a set of criteria $B=<B_1,B_2,\ldots,B_N>$, according to which the performer's conformity assessment is carried out (for example, competence, performance, responsibility, the experience of participation in similar projects, etc.); the higher the score, the more reason to believe that the task will be completed successfully. A membership function can be built on the set of all selected criteria, the values of which for each criterion will express its significance for the solved task of predicting the success of the project.

The task of determining the significance of criteria can be solved by applying the standard algorithm for choosing alternatives (in this case, criteria) based on a fuzzy relationship of preference (Asanov & Myshkina, 2010; Anokhin, 1997; Blyumin & Shuikova, 2001; Kuzmen, 2008). The task is formulated as follows: let $B=<B_1,B_2,\ldots,B_N>$ be the set of selected criteria. It is required to order the elements of the set B according to the significance of each criterion: $\lambda(B_i)$, $i=\overline{1,N}$, N - the number of criteria. In the formation of estimates involved M experts $E=<E_1,E_2,\ldots,E_M>$. To take into account the competence of each expert, the decision-maker (DM), builds a fuzzy preference relation P defined on a set of experts E with membership function $\lambda_P(E_i,E_j)$, the value of which for each pair (E_i,E_j) corresponds to a numerical estimate that the expert E_i is more competent in comparison with the expert E_j, according to the decision-maker.

Then, each expert builds one matrix of fuzzy relationship preferences on the set of all criteria R_l ($l=\overline{1,M}$). If the criterion B_i is more significant than B_j ($i,j=\overline{1,N}$), then write $B_i>B_j$ (not preferable $B_i<B_j$), if the significance of the criteria is

approximately the same – $B_i \approx B_j$, $\lambda_{R_l}(B_i, B_i) = 1$, $i, j = \overline{1,N}$, $l = \overline{1,M}$.

1 Matrices R_l of fuzzy preference relations on a set of criteria can be constructed by the formula (Anokhin, 1997; Blyumin & Shuikova, 2001; Kuzmen, 2008):

$$\lambda_{R_l}(B_i, B_j) = \begin{cases} \lambda_{R_l}(B_i, B_j) > 0 & \text{если } B_i > B_j, \\ \lambda_{R_l}(B_i, B_j) = 0, & \text{если } \mathrm{B_i} < B_j \quad \text{или } \mathrm{B_i} \approx B_j, \end{cases}$$

$i, j = \overline{1,N}$, $l = \overline{1,M}$.

2. The matrix R_l is transformed by entering a fuzzy strict preference relation R_l^S associated with R_l and determined by the membership function (Anokhin, 1997; Blyumin & Shuikova, 2001; Kuzmen, 2008):

$$\lambda_{R_l}^s(S_i, S_j) = \begin{cases} \lambda_{R_l}(B_i, B_j) - \lambda_{R_l}(B_j, B_i), & \text{если } \lambda_{\mathrm{R_1}}(B_i, B_j) > \lambda_{R_l}(B_j, B)_i, \\ 0, & \text{если } \lambda_{\mathrm{R_1}}(B_i, B_j) \le \lambda_{R_l}(B_j, B_i) \end{cases}$$

$i, j = \overline{1,N}$, $l = \overline{1,M}$.

1.1. For each expert, using the following formula, a fuzzy subset of $R_l^{нд}$ non-dominated alternatives is constructed, associated with R_l^S, and including those alternatives that are not dominated by any others, and determined by the following membership function (Anokhin, 1997; Blyumin & Shuikova, 2001; Kuzmen, 2008): $\lambda_{R_l}^{нд}(B_i) = 1 - \max_j \{\lambda_{R_l}(B_j, B_i)\}$, $B_i \in B$.

1.2. A single fuzzy preference relation is calculated taking into account the importance of experts (Kuzmen, 2008):

$$\lambda(B_i, B_j) = \max_{l,t=1...M} \min\{\lambda_l^{нд}(B_i), \lambda_t^{нд}(B_j), \lambda_P(E_l, E_t)\},$$

$i, j = \overline{1,N}$.

1.3. A subset of non-dominant alternatives is determined for a single fuzzy relationship of preference with the membership function $\overline{\lambda}^{нд}(B_i)$, $i = \overline{1,N}$. To do this, repeat steps 1.2–1.3 for the relationship obtained in 1.4.

1.4. The membership function of the resulting subset is (Kuzmen, 2008):

$$\lambda(B_i) = \min\left(\overline{\lambda}^{нд}(B_i), \lambda(B_i, B_i)\right), \quad i = \overline{1,N}.$$

3. To get the final grade c_{lk} of each performer X_k 's job requirements, you can use the formula:

$$c_{lk} = \sum_{i=1}^{N} \frac{\lambda(B_i)}{\sum_{t=1}^{N} \lambda(B_t)} \cdot \mu(x_k^i), \quad (l = \overline{1,m},\ k = \overline{1,n}),$$

where N is the number of criteria; $\mu(x_k^i)$ - a clear assessment of the kth performer according to the criterion, we believe that these estimates take values from 0 to 1; $\lambda(B_i)$ - the significance of the ith criterion.

The closer the score is to 1, the more the contractor meets the requirements of a private project assignment.

4 Description Of The Procedure For Solving The Modified Assignment Problem

To take into account the restrictions imposed on possible distributions, the corresponding terms in the form of penalty functions are added to the objective function, which significantly reduces the distribution estimate in case of failure to fulfill the specified restrictions. We reduce the problem of choosing the optimal distribution of tasks to the problem of finding the minimum of some objective function.

The corresponding objective function for the problem in question can be of the form:

$$F = K_1 \sum_t \sum_i x_{ti}(1 - c_{ti}) + K_2 \sum_t \sum_i x_{ti} z_i + K_3 \left(\sum_k \sum_i \left(\max\left[\sum_{t \in F(k)} x_{ti} \frac{s_t}{p_t} - S, 0 \right] \right)^2 + \sum_t \left[p_t - \sum_i x_{ti} \right]^2 \right)$$

In this expression $x_{ij} \in \{0;1\}$, the variable x_{ij} takes the value 1 - if the ith task is executed by the jth executor, 0 - if the ith task is executed by the j-th executor.

In the objective function:

1. The first term assumes the minimum value if the selected performers most closely meet the requirements of each particular task. When calculating the values of this term (estimates c_{ti}), the algorithm for assessing the compliance of potential performers with the requirements of the private task of the project, given in section 3 of this article, is used.

1. The second term takes the minimum value if the selected performers are least employed in other projects.
2. The third term assumes the minimum value if the total complexity of the tasks performed by one executor at one stage does not exceed the specified value *S*.
3. The fourth term takes the minimum value if each task performs the specified number of performers.

The third and fourth terms of the objective function are penalty functions, the construction of penalty functions is used in the penalty function method used in solving conditional optimization problems in the case where the restrictions have the form of non-strict inequalities and equalities.

Genetic algorithms are often used to solve combinatorial optimization problems, which include the problem under consideration. The genetic algorithm – "is the heuristic search algorithm used to solve optimization and modeling problems by randomly selecting, combining, and varying the desired parameters using mechanisms similar to the natural selection of in nature." The appropriateness of using genetic algorithms in our case is primarily associated with the features of the constructed objective function. To solve the formulated optimization problem, a binary representation of possible solutions (chromosomes) is used, which is quite natural for combinatorial optimization. To solve the problem under consideration, direct coding of possible solutions corresponding to the values of the sought variables x_{ij}, with a code length of *m***n* and a standard genetic algorithm was used.

Other methods can be used to encode possible task distributions, for example, the method described in (Egorova, 2006). This choice will be advisable if the selection of the performer for each task will be carried out not from the whole set of potential performers, but from some part of it. These subsets may include performers whose conformity assessments for a specific task exceed, for example, a certain threshold value. In this case, the number of possible distribution options is significantly reduced.

5 Summary

A computational experiment was conducted to verify the adequacy of the proposed approach. Assessment of compliance with the requirements of tasks, estimates of the degree of employment of performers, the complexity of tasks were randomly generated and optimal distributions were determined using the standard genetic algorithm. The correct distribution of the task was determined on the basis of a complete enumeration of possible solutions. As a result of the experiment, all the obtained distributions were admissible, i.e. satisfying all the conditions introduced by the restrictions, and optimal, or sufficiently close to optimal.

6 Conclusions

The results obtained indicate that the proposed approach is promising and that further research is justified. In the future, it is supposed to adopt the proposed approach in the context of taking into account an even greater number of restrictions and conditions. Another promising task is the automation of the process of obtaining an assessment of the complexity of individual tasks and determining the optimal number of performers for one task. Currently, approaches are being developed that allow for predicting the success of the project to take into account the specifics of individual tasks, their relationships, and value for the project as a whole.

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