MODELS OF PRODUCTION FUNCTIONS OF THE REGIONAL ECONOMY OF RUSSIA (ON THE EXAMPLE OF THE REPUBLIC OF TATARSTAN)

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Abstract: The economic development of any country is determined by the level of production development of its regions. Production functions are an important element of a sufficiently large number of regional development models and allow not only to describe the behavior of the regional economy in an explicit functional form, but also to conduct a quantitative analysis of its effectiveness using the accompanying calculation and analytical apparatus. In this case, the Cobb – Douglas production functions are often used, showing the dependence of the volume of production on the factors that create it. In this paper, the Cobb-Douglas production function is constructed on the basis of statistical data for 2007-2018 from one of the leading regional economies in Russia - the economy of the Republic of Tatarstan. The value of the gross regional product is chosen as the indicator of production volume.

Keywords: Modeling, Regional economy, Gross regional product, Production function, Regression model, Fuzzy linear regression, Triangular Fuzzy numbers, Trapezoidal fuzzy numbers

1 Introduction

The apparatus of the theory of production functions (PF) is widely used by researchers and is considered an effective tool for modeling production processes. It enables to explain the level of total output in terms of the amount of capital and labor expended, the main factors of production. PF has been studied by a sufficiently large number of foreign and Russian scientists at various times, including Kleiner (1986), Makarov (1999), and others.

Currently, PFs are the basis of mathematical modeling of the activity of a wide variety of production structures and systems from individual enterprises and organizations to regions, industries, and the country's economy as a whole (Bessonov & Tsukhlo, 2002; Varian, 2005; Alekseeva & Galiaskarova, 2017; Buravlev, 2012).

In the process of modeling, various types of FSs were considered (linear, power, logarithmic, exponential) and their quality was analyzed according to the accepted criteria. The most famous PF, which is considered classical, is named after the names of its authors - C. Cobb and P. Douglas. Two-factor PFs type Cobb – Douglas are widely used to assess the potential for economic development and assess the prospects for its development. Consideration of certain aspects of the economy of the Republic of Tatarstan (RT) on the basis of PF was carried out in (Parfilova, 2013).

PFs are usually defined as a mathematical model of the phenomenon or process under study, which in the form of an equation or their system describes the dependence of the effective indicator on one or a number of production factors. PF describe the dependence of output indicators of the economic system on input factors (Kleiner, 1986).

As an endogenous variable, GRP is usually considered as an indicator of the volume of product produced in quantitative form, and the exogenous variables are the residual value of fixed assets and the average annual number of employees in the regional economy.

This work is devoted to the construction and analysis of the PF of RT in crisp and fuzzy statements. When constructing a fuzzy PF, the coefficients of exogenous variables were sought in the form of triangular numbers and trapezoidal numbers (Tanaka et al., 1982; Charfeddine et al., 2005). As a result, the endogenous variable (GRP) is also presented in the form of corresponding fuzzy numbers, and interval forecasts are presented as fuzzy intervals.

Here we note the following; one of the first works to study the fuzzy regression problem was Tanaka's work (Tanaka et al., 1982). The work examined fuzzy explanatory variables, crisp regressors, and fuzzy regression coefficients. To find the regression coefficients, the mathematical programming problem is solved. Further development of this approach is presented, for example, in (Tanaka et al., 1989). Currently, a rather large number of papers on fuzzy regression models have been published, devoted to both theoretical aspects and applications of this class of regression models. Among them, we note (Pedrycz, 2015; Tanaka & Ishibuchi, 1991; Redden & Woodall, 1996).

2 Methods

For the study of the economy of RT for 2007-2018, the possibility of applying crisp and fuzzy regression methods is considered for estimating PF in the form of the Cobb-Douglas function (Cobb & Douglas, 1928):

$$GRP = A * K^{\alpha} * L^{\beta},\ \alpha + \beta = n \tag{1}$$

where GRP - gross regional product;
K - value of fixed assets;
L - the annual average number of employees in the economy;
α, β - coefficients of elasticity for factor variables;
A - coefficient reflecting the level of technological productivity;
n- total factor elasticity.
Before proceeding to the construction of the PF, we give the necessary information about fuzzy linear regression.

2.1 Triangular Fuzzy Numbers and Fuzzy Linear Regression

A triangular fuzzy number (A) is a triple (a^L, a, a^R) where the parameters a^L, a, a^R denote, respectively, the smallest possible, the most probable and the largest possible values of the indicator under consideration. A triangular fuzzy number $A = (a^L, a, a^R)$ is called symmetric, if $a - a^L = a^R - a$. A symmetric triangular fuzzy number is given by a triple of the form $(a - d;\ a;\ a + d), d \geq 0$; the number d is called the fuzziness measure of a triangular symmetric fuzzy number. The membership function of a symmetric triangular fuzzy number $(a - d;\ a;\ a + d), d \geq 0$ has the form:

$$\mu_A(x) = \max\left(0{,}1 - \frac{|x - a|}{d}\right)$$

We consider linear fuzzy regression in the form of the basic Tanaka model (Tanaka et al., 1982). It is introduced as a linear fuzzy function:

$$\tilde{Y}_j = \tilde{A}_0 + \tilde{A}_1 x_{1j} + \cdots + \tilde{A}_n x_{nj}, \tag{2}$$

where $\tilde{Y}$ and $\tilde{A}_i$ - fuzzy dependent variable and model parameters;
x_i − independent variable, $1 \leq i \leq n$.
Here for each j, an inequality is performed $\mu_{Y_j}(y_j) \geq h$, where h is some predetermined threshold value in advance.
With this statement, one speaks about the problem of fuzzy linear regression with a threshold value of h. Moreover, each fuzzy parameter $\tilde{A}_i = (a_i - d_i, a_i, a_i + d_i)$ is described by a symmetric triangular membership function, which consists of a fuzzy center a_i and a fuzzy half-width d_i. Then Y_j has the following form:

$$Y_j = (z_j - r_j, z_j, z_j + r_j),$$

где $z_j = a_0 + \sum_{i=1}^{n} a_i x_{ij}$, $r_j = d_0 + \sum_{i=1}^{n} d_i |x_{ij}|$.
The total measure of fuzziness is calculated by the formula:

$$r = \sum_{j=1}^{m} r_j = m\, d_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} d_i |x_{ij}|.$$

The problem of fuzzy linear regression is transformed to the following linear programming problem:

$$r = \sum_{j=1}^{m} r_j = m\, d_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} d_i |x_{ij}| \to min;$$
$$y_j \geq \sum_{i=0}^{n} a_i x_{ij} - (1 - h) \sum_{i=1}^{n} d_i |x_{ij}| \quad , \quad = 1{,}2, \ldots, m \quad ;$$
$$y_j \leq \sum_{i=0}^{n} a_i x_{ij} + (1 - h) \sum_{i=1}^{n} d_i |x_{ij}| , = 1{,}2, \ldots, m ; \tag{3}$$

$d_i \geq 0$, $i = 1,2, \dots, n$.
This method of constructing a fuzzy linear regression is used to evaluate the model of economic growth in the region in the form of a fuzzy PF Cobb-Douglas:

$$GRP = A_0 * K^{A_1} * L^{A_2} \quad (4)$$

Model parameters A_0, A_1, A_2 – fuzzy symmetric triples of numbers:

$A_i = (a_i - d_i, a_i, a_i + d_i)$

where a_i – the most probable value of the coefficient; d_i – width fuzzinesses of the coefficient.
We linearize function (4) by taking the logarithms. As a result, we get a log-linear model of the form:

$$ln\, GRP = lnA_0 + A_1 lnK + A_2 lnL$$

This dependence can be estimated using the fuzzy regression method (Tanaka method). The resulting fuzzy coefficients of the model are determined by triangular fuzzy numbers. After moving to the source variables, we have the following model:

$$\widehat{GRP} = e^{A_0} * K^{A_1} * L^{A_2}$$

1.2 Fuzzy regression analysis using trapezoidal fuzzy numbers.
We use the h-cut method in our work (Charfeddine et al., 2005). A fuzzy function of level $\tilde{f}$ is defined as four levels of crisp functions: f_a , f_b , f_c , f_d. For consistency's reasons, these four functions cannot intersect in the input area given by:

$[X_{min}, X_{max}] = ([(x_1)_{min}, (x_1)_{max}] \times \dots \times [(x_N)_{min}, (x_N)_{max}])$:
$\forall x \epsilon\, [X_{min}, X_{max}]$, $f_a(x) \leq f_b(x) \leq f_c(x) \leq f_d(x)$.
To determine the function of the extreme level f_a *and* f_d the Tanaka model is used, taking into account the resolution (3) for *h* = 0:

$$f_a(X) = \sum_{i=1}^{n} a_i{}^* x_i - \sum_{i=1}^{n} d_i{}^* |x_i| \, , \quad (5)$$

$$f_d(X) = \sum_{i=1}^{n} a_i{}^* x_i + \sum_{i=1}^{n} d_i{}^* |x_i| \quad (6)$$

The f_b and f_c levels are determined by solution (3) for *h* selected from the interval [0,1[.
We turn to the model of the fuzzy production function (4) in the case when the model parameters A_0, A_1, A_2 are fuzzy trapezoidal numbers. We preliminary linearize it by taking the logarithms:

$$ln\, GRP = lnA_0 + A_1 lnK + A_2 lnL$$

where $A_i = (a_i, b_i, c_i, d_i)$

We estimate this dependence using the fuzzy regression method (the method using the *h*-cut). After the moving from the estimated log-linear model to the initial PF, we obtain a model of the form:

$$\widehat{GRP} = e^{A_0} * K^{A_1} * L^{A_2} \, ;$$

where $A_i = (a_i, b_i, c_i, d_i) =$

$$= (a_i - d_i, a_i - (1-h)d_i, a_i + (1-h)d_i \, , a_i + d_i).$$

In this work, the series of variables are obtained from official information posted on the website of the Federal State Statistics Service (https://www.gks.ru/regional_statistics), the data are presented in table. 1.

Table 1: Initial data for building a regional Cobb-Douglas PF

Years	GRP, (RUB.Million)	Cost of fixed assets, (RUB. Million)	Employed in the economy, (thous.persons)
2007	757401.4	1586177	1812.926
2008	926056.7	1802843	1871.006
2009	885064	2132421	1823.079
2010	1001623	2526863	1899.246
2011	1305947	3461464	1939.862
2012	1437001	3110418	1966.287
2013	1551472	3342559	1961.421
2014	1661414	3431206	1975.912
2015	1867259	3921931	1980.164
2016	1933092	4256272	1980.758
2017	2139810	4658900	1968.186
2018	2469217.4	5033940	1963.493

3 Results and Discussion

Consider the framework of the work, constructed crisp and fuzzy PF RT and analyze their quality indicators. In the environment of the Gretl econometric package, an inhomogeneous PF Cobb-Douglas RT is constructed in a crisp statement using OLS for evaluating the log-linear model. The log-linear model has the following format:

$$\widehat{ln\, GRP} = -12{,}355 + 0{,}86lnK + 1{,}806lnL$$

The model is characterized by the following: a high multiple determination coefficient R^2=0,948, statistical significance in general by the *F*-test at the significance level α=0,01, but the free term and regression coefficient for *lnL* are insignificant even at the level α=0,1. This is a sign of multicollinearity factors. It should also be noted to the short sampling. In such conditions, it is not possible to obtain qualitative estimates of the elasticities of the PF were obtained by the moving from the log-linear model:

$$\widehat{GRP} = 0{,}0000043K^{0{,}86}L^{1{,}806}$$

When the PF is transformed to a linear form the transformed dependent variable undergoes transformation, therefore, OLS-estimates of model parameters are ineffective. Their values may also be biased. These problem points are solved by nonlinear OLS. The PF is estimated using this method in the Gretl environment has the following form:

$$\widehat{GRP} = 0{,}00026K^{1{,}032}L^{0{,}918}$$

However, this model, despite a high level of data fitting and significance in general, has insignificant free term and regression coefficient for the factor *L*. This leads to low quality estimates of the elasticities of the PF. Taking into account in the model the time factor in the form of a multiplicative term of the form e^{pt} does not improve the situation.

Only the one-factor PF is statistically reliable. The model by nonlinear OLS has the following form:

$$\widehat{\mathrm{GRP}} = 0{,}1269\mathrm{K}^{1{,}08}$$

Model quality indicators: R^2=0,954, *MAPE*=7,198%. The regression coefficient is significant at the significance level α=0,01, the free term A=0,1269 is insignificant. It should be noted that here *A* – coefficient that takes into account the average influence of factors that are not included in the PF equation. The confidence interval for the coefficient of elasticity by the capital factor with a 95% reliability is from 0,895 to 1,274. There is no autocorrelation in the remnants of the model. We present the results obtained when constructing a PF based on the fuzzy regression method in the MS Excel table processor environment. A fuzzy logline model was constructed using the Tanaka method, and its free term and coefficient for *lnL* were equal to zero. After moving to the original PF RT, the following fuzzy one factor model was obtained using triangular numbers of the form:

$$\widehat{GRP} = K^{(0,935\ ;\ 0,944\ ;\ 0,953)}$$

In this model, a very high value of R^2= 0,99 (calculated using defasified predicted GRP values). The free term A=1, and the coefficient of elasticity by labor L is 0 (labor factor does not significantly affect output). Coefficient $\tilde{A}_1 = (0,935;\ 0,944;\ 0,953)$ −fuzzy coefficient of elasticity for fixed assets. Consequently, the increase in fixed assets by 1 % corresponds to the average increase in output in the range from 0,935 % to 0,953 % with the most expected 0,944 %.
A fuzzy PF using trapezoidal fuzzy numbers (h-cut=0.8) has the form:

$$\widehat{GRP} = K^{(0,935\ ;\ 0,942;0,946\ ;\ 0,953)}$$

In the model, the value of the multiple determination coefficient is high R^2= 0,99. The model shows that a 1% increase in fixed assets corresponds to an average increase in output in the range from 0,935 % to 0,953 % with the most likely range from 0,942 % to 0,946 %. The centroid of the trapezoidal fuzzy elasticity coefficient for fixed assets is equal to 0,944 using the center of gravity method (Chernov, 2018).

4 Summary

As a result of the analysis of the economy of the Republic of Tatarstan for the period 2007-2018, it is empirically shown that crisp and fuzzy adequate and statistically reliable two-factor PF type Cobb-Douglas cannot be constructed. PFs with such properties turn out to be one-factor.

The RT economy can be adequately described by power one-factor PFs with high values of R2>0.9 and MAPLE<8%, based on both crisp and fuzzy approaches. In all significant models, only the capital factor turned out to be significant, while there is a decreasing return on the capital factor. The growth of the labor factor does not naturally lead to a corresponding increase in GRP.

5 Conclusions

In conclusion, we note the following. Studies have shown that the use of classical PFs type Cobb-Douglas with constant returns is unsuccessful for describing the economy of the Republic of Tatarstan in the considered time range. PF by the method of nonlinear OLS leads to a negative value of the coefficient by the labor factor, which contradicts the economic essence of the problem. Crisp and fuzzy adequate inhomogeneous PFs turn out to be one-factor with respect to the capital factor. The introduction of a time trend also does not significantly affect the explanatory properties of the model. Given the confidence interval for the elasticity by the capital factor of a crisp PF, we can conclude that a fuzzy PF allows us to draw more adequate conclusions regarding the economic growth of RT. The conducted empirical research shows that in the economy of the RT during the analyzed period, an intensive growth is observed.

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