BEST PRACTICES FOR IMPROVING SPATIAL IMAGINATION IN MATHEMATICS

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Abstract: In this study, we collected best practices and tools for the development of spatial imagination. We divided tools into 3 groups: planar aids, spatial aids, and board games. In the case of planar aids, we present the Tangram puzzle, the Geoboard, the Indian mosaic and the Poliomino. As for spatial aids, we analysed the Polydron, the Geomag, Pop-up projects, the Zometoll, the 4D Frame, and the Jomili as suitable development tools. It is difficult to find a suitable development game among board games, because they are explored insufficiently. We recommend four games, the Ubongo, the Q- bitz, the Rumis and the Blokus. In this study we would like to eliminate the shortcomings in the teaching of mathematics, because these best practices, which develop spatial imagination, are not sufficiently known in Slovakia.

Keywords: Spatial Imagination, Manipulation, Game, Collection, Development.

1 Introduction

Developing the spatial imagination of schoolchildren is an important task in the educational process, especially with regard of the practical application of acquired competencies in real life. Spatial imagination is “the sum of abilities related to reproductive and anticipatory, static and dynamic ideas about shapes, their properties and mutual relations between geometric shapes in space” (Molnár, 2004, p. 7). J. Piaget and V. Repáš state that “there are certain time periods that are particularly favourable for the development of spatial imagination. When these periods are missed, one loses the opportunity to develop his/her abilities to the level given by genetic predispositions” (Hejný et al., 1990, p. 368). Geometric spatial imagination is the ability to perceive

- geometric formations and their shape, dimensions and location in space,
- formations in different positions, different from the original position,
- deformation of dimensions, structure, etc.,
- surface representation and oral description of bodies,
- creating a spatial model based on the image shown in the plane.

Based on the results of research conducted in 2012, it was also proved that the development of spatial imagination is helped by our own experience gained in the development of fine motor skills and psychomotor skills (Gabajová, 2012).

Perný (2004) also deals with the development of spatial imagination, the so-called spontaneous stereometrics, i.e. the creation of basic concepts – ideas about bodies and their properties. He identifies four phenomena that are important for the development of spatial imagination:

1. The language phenomenon – refers to communication, e.g. understanding and verbal determination of the front and back side of the cube, the direction of rolling and the forward and backward movement.
2. The phenomenon of the concreteness of the idea – examines what type of cube model students imagine, e.g. more often they imagine a complete, opaque model than an empty wireframe.
3. Kinaesthetic phenomenon – describes the accompanying movement phenomena that students induced when they solve a problem, e.g. when the cube turns in the mind, whether it is related to the hand movement signal.
4. The phenomenon of the use of regularities – certain regularities occur in certain situations, e.g. the sum of the values on the opposite dice sides equal to 7.

In recent years, the evaluation of spatial imagination has been carried out mainly in the context of STEM (science, technology, engineering, mathematics) and the results confirm their close connection. (Cheng and Mix, 2014; Newcombe, 2013; Uttal and Cohen, 2012).

The manipulation activities applied in the teaching of mathematics should be present at the forefront of teachers' work, as it plays a crucial role in the effective acquisition of knowledge and in the expansion of the range of knowledge. Active cognition develops memory, understanding the context, constructive thinking, individual abilities, positive qualities and ultimately motivates students. Spatial concepts, knowledge and attitudes are formed by the students' own experiences. The construction of various geometric bodies develops spatial imagination as well as constructive thinking of students. The development of manipulative activity in the teaching of mathematics began after the publication of the results of Piaget's research. His research clearly demonstrates: “The content of thinking cannot be considered a static reflection, but should be understood as an active intellectual schema that develops during a particular activity, it is mastered, regenerated by repetition, differentiated, optimized the content, and regenerated.” (Czeglédy and Hajdu, 1982).

In connection with the system and development of cognitive abilities, Nagy (1998) points out that during the creative process and problem solving, a new product will be created, and thus new knowledge. In his study, he defines three inherited forms of knowledge acquisition: (1) exploration, (2) testing, and (3) play. In its interpretation, play essentially represents “simulating behaviour”, “the independence of cognitive competence, which means direct action with signs and symbols” (Nagy, 1998, p. 11). Game could play a more important role in the learning process, as scientists are also showing a growing interest in game-based learning, explore and discover ways in which different types of games can be integrated into the school context. The right game, applied in the right place and at the right time in math class, can make learning easier and more efficient in the long run. It is important to recall Fröbel’s pedagogical activity in connection with the introduction of games into the teaching process. In addition to developing a system of gaming devices called “gifts”, he also developed strict instructions and methodological descriptions of their use. “In Fröbel’s kindergarten, the acquisition of knowledge did not take place through speech and intellectual exercises, but through the active activity and creation” (Szabolcs and Réthy, 1999, p. 364). Learning is born through games from the integration of play in education, which can significantly increase interest in learning (Chung, Yen-Chih , Yeh and Lou , 2017). Learning by playing provides a new interface for pupils and teachers and increases bias, and motivation to learn. The content of game provides new knowledge, even development of skills is influenced by the gaming the experience itself (McFarlane, Sparrowhawk, and Heald, 2002).

2 Spatial aids

The Polydron geometric construction system is an English invention that has existed for more than 30 years. Technical designer Edward Harvey worked on a project in 1970, laying the foundations for this internationally recognized system. In 1982, Edward Harvey’s son founded POLYDRON company, which quickly gained international recognition. Roman Harvey began producing safe, strong and easy-to-use geometric shapes from high-quality plastics. From them it is possible to model two-dimensional and three-dimensional geometric models, which are especially suitable for recognizing the properties of geometric bodies. Older children can use the kit, e.g. for modelling polygons (4-side, 6-side, 8-side, 12-side, 20-side) for modelling other bodies. Modelling of these bodies brings the possibility of new discoveries (Figure 1).

The Geomag is a set that was originally born from an Italian idea and offers geometric modelling for children of all ages. The physical and technical essence of the kit is magnetism. The model of the geometric point is a metal sphere with a diameter of about 1 cm and the straight model is represented by rods of various lengths made of plastic or metal, at the ends of which are placed super strong magnets (Figure 2). It follows from the above that balls and rods can be joined into different planar and spatial shapes. This set is suitable for the development of geometric spatial perception.

Figure 1: Geometric shapes of the Polydron

The experience of Fiala (2013) shows that mathematical education needs to be revived and active learning should be given priority, including in the teaching of geometry. In his research he successfully uses the Geomag, the Zoometole, and the Polydron sets. Previous research has shown that block games are particularly important for the development of spatial skills and understanding in children. Based on case studies by Ness and Farenga (2007) showed how children develop their geometric, spatial and scientific skills in free play with blocks. It was found that when children were engaged in block play, their spatial and geometric concepts as well as architectural principles were updated, and that playing with blocks had a positive effect on children's mathematical behaviour in general.

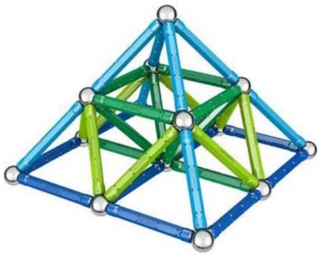

Figure 2: The Geomag mode

Probably less well known is the application of Pop-up projects in mathematics. Their goal is to use the paper and scissors to develop the spatial imagination from planar formations to spatial formations. These formations are also often referred to as "living books". These books were published in the 18th century and were used to entertain children. The first real Pop-up book "Red Riding Hood" was published in 1855 in London. German publishers also gradually joined, especially during the period of rapidly developing colour printing and book production. Finally, in 1925, the first theories emerged that transformed sheets of paper into three-dimensional objects.

Pop-up books are no longer published just for fun but pursue serious educational goals. Paper-cutting and folding techniques allow mechanisms to fold books into three-dimensional shapes (Figure 3). Geometrically, movement, rotation, but also deformation of parallel planes is possible. The resulting shapes differ from the original not only in their third dimension, but also in that they are created in a dynamic way. Imagination is essential for the successful creation of auto-opening products that involve linking the design phase and the implementation phase with the correct prediction of the outcome. Stages of creating a pop- up model:

1. The project is made on a cardboard sheet.
2. Cut lines and bending lines must be marked separately.
3. The sheets are cut and bent.

The observation of manufactured products makes it possible to discover the transition from planar geometry to spatial geometry, and in the meantime the inverse mechanisms of this transition appear to us. For example, in a cube, we can observe that the sections are parallel to each other, as well as the bends; and the cuts and bends are perpendicular to each other.

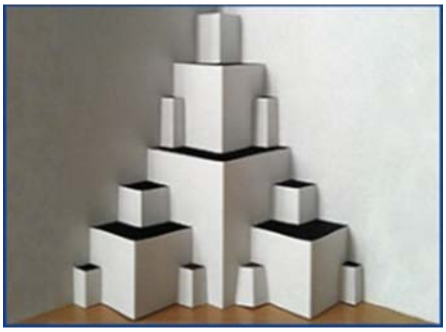

Figure 3: Pop-up model

The Zometoll model set is a unique tool for learning, creating and playing together, and is even able to accomplish this complexity in a simple way. The game with elements of different colours and shapes surprisingly provides an opportunity to get to know the properties of two-, three- or even multidimensional spaces. It develops imagination about plane and space, and planning skills (Figure 4). This creative geometric set allows you to create a wide range of spatial models.

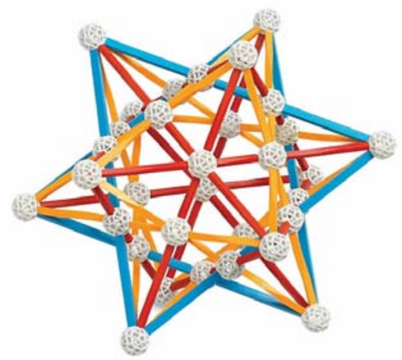

Figure 4: The Zometoll model

The 4DFrame is a new generation of educational tools invented and developed by Ho Gul Park in South Korea. Although his concept and function are simple, the potential for their use is unlimited. Coloured pipe pieces of different lengths are attached and connected together with connections of different shapes, which allow the creation of different types of formations and shapes (Figure 5). It is a useful tool for recognizing basic geometric formations and shapes. Children can even assemble various models with moving parts, such as cars, airplanes, windmills and water wheels. The set provides options for individual and group activities. Groups can learn to work together or discover their own ideas.

The KomTek, based in Järfalle, is a Swedish science and technology centre focused on activities, which regularly uses the 4D Frame tools and collaborates with local schools to develop their applications. Their research has shown that an educational program using the 4D framework improves spatial feeling and mathematical creativity in primary school students (Lee, 2013).

The Jomili set was invented by László Lukovics, who was inspired by the works and creative methods of the painter Victor Vasarely. From the elements of a set of eight differently painted cubes and blocks, it is possible to create a myriad of images and spatial creations (Figure 6). The game develops motor and cognitive skills, dexterity, spatial imagination, creativity and logical thinking. It is not yet widespread in the educational process abroad nor in Slovakia.

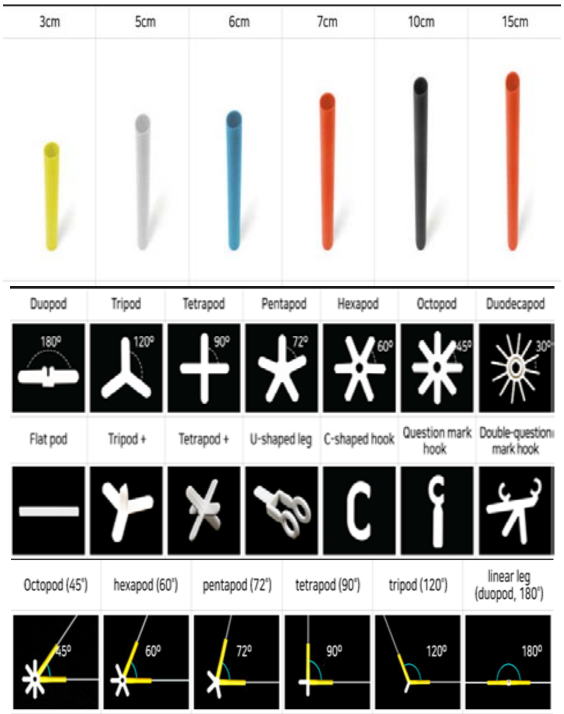

Figure 5: The 4D Frame part of the game

Historically, the combination of origami and mathematics began with the book "Geometric Procedures in Folding Paper" written by the Indian mathematician Tandalam Sundara Row (Row, 1917). In the book, he presented a new and very simple way of realizing geometric constructions. Instead of drawing classic lines and circles, he used paper puzzles (Figure 7). Some math teachers have used origami for long to make their lectures or practices more attractive, and lively.


Figure 6: The Jumili building

There is a lot of literature on this topic, from classical geometric techniques in folding paper to Hull's recent book (2006) entitled "Origami, Activities for Exploring Mathematics", which deals with folding paper as a playful form of learning mathematics.

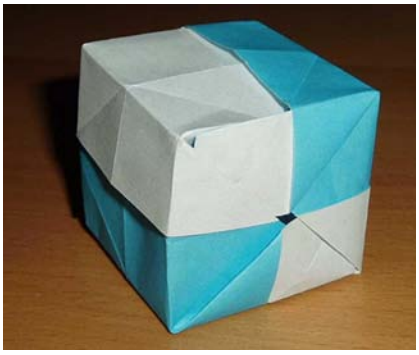
Figure 7: Geometric shapes with paper folding

We can also model geometric bodies with mallets and straws, which we all know, although we do not use these tools in mathematics lessons often enough to improve the geometric imagination. Based on our own experience, we can say that they are excellent for modelling of planar geometric shapes and for the construction of regular bodies, blocks and pyramids within spatial geometry (Figure 8). The elements can be easily inserted into each other and can be used repeatedly.

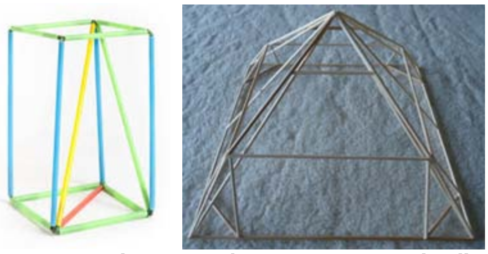
Figure 8: Spatial geometric shapes using straws and mallets

3 Board Games

Previous studies have shown many times that game-based education can also be effective in developing spatial skills. Positive effects of the Lego, the Rubik's Cube, the Pentomino, and the Tangram for the development of these skills are known. However, the effects of board games on spatial imagination have rarely been studied, so there is little research to help choose the right development board game. In various non-scientifically based collections, board games were collected that best develop the spatial imagination. We also present board games that could be interesting for senior students in primary schools. They were found in the lists of board games that best develop spatial imagination and for which positive results have emerged in the development of spatial imagination in scientific experiments.


Figure 9: Components of the Ubongo board game

In the Ubongo game, players must use 12 cards to create and cover an empty space on the task board. The game improves the spatial and logical abilities of the players, as they must manipulate, rotate and compare different shapes, then choose the right ones and interpret the diagram according to the assignment (Figure 9). The best use of the Ubongo game is to develop the ability to know and identify. Because the player must think about rotating and turning cards, this greatly improves the ability to identify spatial shapes (Chung et al., 2017).

The Q-bitz is a visual spatial logic set consisting of four wooden logic sets, each containing 16 pieces of sampled cubes. The cubes of the set have the same design, differing only in colour. The set contains 80 logical puzzle-cards of various difficulty. The Q-bitz puzzles are excellent for developing the visual abilities of distinguishing sampling and spatial thinking abilities. During the game, the pattern shown on the puzzle cards must be stacked using 16 dice so that the sides of the dice are placed on the correct side, as each of their sides covers a different pattern. The puzzles are excellent for teaching concepts of symmetry.

Figure 10: The Q-bitz board game and sampled dice

The Rumis game was inspired by the image of the ruins of the Inca temples including pyramids, tall towers, staircases and various walls. Players must place coloured shapes similar to the colour-painted Tetris wood element so that our own building blocks touch each other during the game (Figure 11). This game thoroughly improves the spatial imagination, as we are still working to place as many of our own elements as possible, while preventing the opponent from placing his elements, which means thinking about possible moves of the opponent, which also leads to tactical considerations. At the end of the game, the one who occupies a larger area wins, as seen at the end of the game from above (Chung et al., 2017). The Rumis game won the award Mensa Select Winner in 2004.

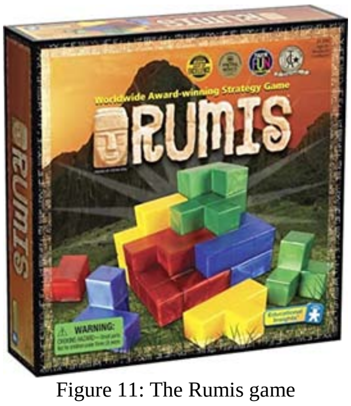

Figure 11: The Rumis game

The Blokus is partly similar to the well-known game Go. Players take turns placing Tetris-shaped pieces on the board, starting from the corner. Each new piece must touch the corner of a previously placed piece of the same colour. The purpose of the game is to occupy as much space on the board as possible, and thus prevent the expansion of your opponents. The game ends when we can no longer place more pieces and all players have been blocked. This game won the Mensa Select Award in 2003. In terms of the development of spatial imagination, the set contains 4 x 21 different Tetris-like shapes of the same colour. These shapes need to be identified, compared and analysed during the game in order to create a suitable game strategy (Chung et al., 2017). There is also a 3-dimensional version of the Blokus, the Blokus 3D, which is almost identical to the Rumis game described above.

Figure 12: The Blokus and the Blokus 3D social set

4 Conclusion

We have found several remarkable research findings on the use of each of the tools described, but we have not found such a set of manipulation tools that improve spatial imagination. We wanted to partially eliminate this shortcoming and we tried to collect various tools that can be helpful to our colleagues and mathematics teachers. Through this study, it was found that there are number of tools that effectively develop spatial imagination that have been proven by research. The development of spatial imagination using these tools could therefore work more efficiently and thoroughly. Greater presence of creative and constructive activities in the educational process can also contribute to the realization of many other educational goals. The application of these best practices in the teaching of mathematics in Slovakia is nevertheless lacking, so we can talk about a shortcoming that needs to be remedied. Educators use these tools less often, mainly due to lack of time and lack of tools. Research has clearly shown that constructive games have a positive effect on the development of spatial imagination, and also on the development of mathematical and scientific performance (Richardson et al., 2011).

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