RESULTS OF ANALYSIS OF TASKS SOLUTIONS FOCUSED ON SELECTED ELEMENTS OF PROPOSITIONAL LOGIC IN THE CONTEXT OF SCIENCE AND MATHEMATICS SUBJECTS IN THE POST-FORMAL STAGE OF COGNITIVE DEVELOPMENT

[a]KATARÍNA SZARKA, [b]ZOLTÁN FEHÉR, [c]LADISLAV JARUSKA, [d]EVA TÓTHOVÁ TAROVÁ

J. Selye University, Bratislavská cesta 3322, SK-94501 Komárno, Slovakia
email: [a]szarkak@ujs.sk, [b]feherz@ujs.sk, [c]jaruskal@ujs.sk, [d]tothovatarovae @ujs.sk

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Abstract: The current study presents the results of empirical research, which focuses on analyses of the solution of tasks containing the selected elements of propositional logic formulated in different subject-specific contexts. The focus group of our interest was the university students. As a research tool, we used a knowledge test. Our research aimed to investigate the results of the respondents (N = 497) depending on the selected aspects (students' performance according to the students' study programme and their success in solving individual tasks depending on the subject-specific context, gender, age, type of secondary school where they graduated). The results show that students' chosen orientation in higher education is typically related to the successful application of formal logic and its subject-specific contexts. Furthermore, the results confirmed that secondary grammar school education still provides better preparation in formal logic than other types of secondary schools.

Keywords: formal logic, post-formal cognitive level, scientific and mathematical tasks, higher education.

1 Theoretical background

Thought operations play a crucial role in an individual's cognitive development. They form the basic cognitive apparatus of a person. The fundamental pillars of this apparatus are four skills: the skill of arrangement, combinatorial, logical skill, and skill of proving. The skill of arrangement is the one that develops first in children, then the combinatorial skill and logical thinking develops, and most recently, the skill of proving. Why is logical thinking important? As a result of the absence or insufficient logical thinking, pupils have problems with comprehension and interpretation of the text and general information processing, which makes it difficult for students to learn.

Understanding how we think, gain knowledge, and learn is currently being explored from various aspects - from genetic factors, epistemology, philosophy of thought, language structure, educational theory, and other areas. Experts` debate is the possibility of identifying general aspects of thinking in various contexts. There are opinions according to which there is a general ability to think that can be used or applied in different contexts (Ennis, 1989, 1996). On the other hand, thinking proponents are always context-specific (McPeck, 1990), and thinking is applied as part of situational learning (Lave, Wenger, 1991). Theories of education tend to look at this issue from the point of view of learning knowledge or the development of knowledge, from a genetic point of view, based on Piaget's generally accepted theory of cognitive development.

The person`s cognitive development is connected to language acquisition (Clark, 2003). In the works of Piaget and Vygotsky, we know that although the roots of the genesis of thought and language are different in the development of the individual, their connection and relationship strengthen with age. Speech, therefore, becomes an instrument of thought, expressed in words.

Piaget characterizes the relationship between language and thinking as follows:

- the system of rules of logical thinking is not innate but evolves gradually,
- the language itself is not strictly tied to logic,
- the logical nature of our thinking depends on the properties of thought operation structures that gradually evolve during the development of the individual.

It is also clear that language significantly influences thinking through communication. In everyday life, through language and speech, we use many logical structures created by conjunctions, which are also operators of propositional logic. Behind the logical structure of speech lies an important logical content. If a person does not have the basic apparatus of propositional logic and does not understand logical structures, he does not understand its content. Research studies show that language slightly affects thinking in other ways than communication, such as mathematical reasoning and spatial navigation (Bloom, Keil, 2001). Language plays an exciting role in how we think beyond its role in communicating information. "*The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve*." (Wigner, 1960)

Through mathematics as a discipline and as a school subject, students' thinking can be shaped to apply the mathematical knowledge they acquire to other subjects and solve everyday problems outside school. Mathematics offers a systematic approach to solving various problems, and this apparatus is suitable for modeling natural and social phenomena (Csapó, Szendrei, 2011). For example, the mathematical background of deductive reasoning is classical formal logic, where the emphasis is primarily on the formation and interpretation of complex statements from elementary judgments using conjunctions ("and", "or", "if..., then" and "then and only if..., if") as linguistic elements. Equally important is the correct interpretation and use of quantifiers often used in everyday speech, such as the linguistic schemas "all" and "there are such" and their synonyms (Vidákovich, 2013).

Usually, when we talk about thinking, many people associate it with mathematical thinking. Moreover, mathematical thinking tends to be reduced to numerical and arithmetical problems. When we think of mathematical thinking in the context of science, we immediately think of calculations in chemistry, physics, or biology. Thinking in the context of science must be understood in a much broader sense, just as in the case of mathematical thinking, we must think of more than just doing numerical calculations.

Since its founder Aristotle, the science of correct reasoning, i.e., drawing conclusions, and driving consequences from given assumptions, has been called logic. Mathematics as a science is based on this logical basis, as well as the teaching of mathematics, one of the main goals of which is the development of logical thinking. At the same time, in contrast to classical logic, modern logic is strongly influenced in its form and methods, especially by mathematics, and is called mathematical logic. Its basic parts are propositional logic and predicate logic, some of which we acquaint students with a didactically appropriate and accessible form in the introduction to mathematical logic at the beginning of high school mathematics (Polák, 2014).

The development of formal reasoning ability should be a significant priority also in science education (Lawson, 1982). There is a direct link between formal thought and integrated processes such as identifying and controlling variables and hypothesizing. It is reported that formal reasoning ability was the strongest predictor of process skill achievement and retention (Tobin, Capie, 1982). Teaching biology, physics, and chemistry provide an excellent opportunity to develop thinking. Advanced thinking skills are needed to understand and apply basic concepts and recognize connections (Radnóti, Korom, 2020).

Research shows that speech has the most significant effect on developing thinking at the stage of formal operations (in children between the ages of 11 and 15) (Csapó, Vidákovich, 1987). It is interesting that in children, the grammar development precedes the development of logic: expressions such as "because," "when," "but," and "on the contrary" are used by the child before he knows and understands their real logical meaning. According to Chomsky (2006), when creating a sentence from an idea, we perform a sequence of mental transformations, but the structure of natural language is not based on logic. In order to examine logic independently of language, we need to "cleanse" the logic of language ties. For such purposes, propositional logic is optimal, independent of natural language or interpretation's manner and psychological factors.

Propositional logic formalizes the language through which we formulate mathematical statements; it sets out the rules by which we can infer new statements from statements; it analyzes the forms of propositional structures and develops methods of proof. By Piaget (1970), signs of logical thinking in formal logic can already be found in the preoperative developmental stage of a child (2-7 years). The stage of specific operations is an essential milestone in logical thinking. In addition to the typical characteristics of this stage, such as logical thinking about objects and events and understanding the constancy of number, quantity, and mass, there are also elements of combinatorial thinking. According to Piaget and Inhelder (2013), the recognition of the terms "possible" and "true" leads to the development of two-variable logical operations, which is a prerequisite for the use of formal logic and thus formal thinking. For teenagers, operations of a hypothetical-deductive nature at the verbal level make it difficult to use concepts and expressions instead of natural objects and build a "new" logic - a system of propositional logic and mainly apply it at the verbal level in various contexts is cumbersome. Piaget also calls this stage a period of propositional logic (Piaget, Inhelder, 2013).

Current trends in the development of logical thinking focus more on developing individual elements of operations supporting logical thinking than on the holistic development of general logical thinking (Csapó, 2018). It turns out that the development of logical thinking skills is effective if educational activities aimed at applying individual logical operations are adapted. Namely, their proper use is a prerequisite for managing thinking and its improvement. It is possible to think logically without knowing the individual elements of formal logic. Many studies (Csapó, 2018; Carlsson et al., 2015; Csapó, Molnár, Nagy, 2014) currently address the diagnosis of cognitive abilities, including the ability to think, and the issue of their development. Studies that approach thinking through logic manifested in the language and statements have a more extended history than the early 20th century. It has three main areas that roughly correspond to traditional chapters of formal logic statements (Nitta, Nagano, 1966; Braine, 1978), quantifiers (Revlis, 1975; Johnson-Laird, 1983, 2005), and transitive deduction (Huttenlocher, 1968; Clark, 1969).

Sinnott (1988) examined logical thinking with the tasks of Piaget among adults in the postformal stage of cognitive development and outlined the possibilities of developing logical thinking in adults. These alternative models ultimately claim that the human mind has more than formal or logical rules. As Johnson-Laird (1983) pointed out, the most significant problem is that people make mistakes. They draw invalid conclusions that should not occur if the deduction follows mental logic. The basic premise of any formal logic is that deductions are valid based on their form, not their content. If a derivation rule is in mind, it should apply regardless of the proposal's content (Lourenço, 1995). Carey (1985) pointed out that there are apparent differences in the thinking of children and adults regarding specific knowledge - children are newcomers in all areas of thought, while adults show thinking skills.

2 Aim of research

The main objective of our research was to assess the level at which students can use selected elements of formal logic. The empiric research aimed to examine the results of solving a set of tasks focused on elements of formal logic concerning to the nature of students' studies. In the research, we worked with an available sample of university students from two faculties, the Faculty of Education and Faculty of Economics and Informatics, whose study programs are oriented diametrically differently. While the Faculty of Education (FEd) mainly offers study programs with a more humanistic and social science orientation, the Faculty of Economics and Informatics (FEI) focuses on the technological and economic areas of STM (science, technology, mathematics). Our research primarily want to compare students` results from these two different oriented study programs. Accordingly, we formulated four research questions and hypotheses:

Q1: Are there differences in students' performance in the formal logic test according to the students' study programme?
H1: We assume that FEI students achieve better results in solving the tasks because they have more subjects requiring logical thinking taught at FEI (mathematics, computer science).
Q2: Within each task-group containing different elements of propositional logic, is there a difference in the success of each subject-related task in terms of study programmes?
H2: We assume that FEI students achieve better results compared with the students of FEd.
Q3: Is there a difference in the students' performance of task-group solutions in terms of gender and age?
H3: We hypothesize that there is no significant difference in the success of task solutions in terms of gender and age.
Q4: Is there a difference in the test results according to the type of secondary school which the students finished?
H4: We assume that students who finished secondary grammar school achieve better overall results than other secondary school students. Moreover, we think that the success of secondary grammar school graduates is independent of the study program in which they are currently studying.

We see the reason for our assumption in the higher standard of grammar school education, and its content structure provides a better basis for students to achieve in the logic test.

3 Materials and methods

The research involved 536 students, 184 persons from the Faculty of Education (FEd) and 313 from the Faculty of Economics and Informatics (FEI). A total of 536 participants completed the questionnaire, and after validation of the data, 497 respondents were included in the study. The sociodemographic characteristics of the participants are reported in Table 1.

Table 1: Sociodemographic data of respondents

Variable	Frequency (*n*)	%
Sex		
female	310	62.6
male	185	37.4
Age		
18-25	401	81.0
> 25	94	19.0
Study programme		
FEI – management	241	48.5
FEI – applied informatics	72	14.5
FEd – Pre-school education, Primary school teacher	158	31.8
FEd – Teacher training	26	5.2
Type of secondary school		
Secondary grammar school	199	40.1
Secondary school	297	59.9

3.1 Measuring instrument

The method used to achieve our research aims was a single-group quasi-experiment performed in February - April 2021. As a research tool, the survey was composed of a sociodemographic part, including sex, age, study program, type of secondary school education, and a knowledge test. The knowledge test included a set of 15 tasks from biology, physics, chemistry, mathematics, and the context of everyday life. The tasks were selected according to their nature of propositional logic and further grouped into three groups. All three groups included 5-5 tasks, one from each subject-specific context. In group A, there were tasks in which we focused on monitoring the understanding and correct use of quantifiers: "everyone," "at least," "just," "most," and "none." In group B, there were tasks in which we surveyed whether the respondents knew the correct formulation of the negation of the statement. Group C included tasks on the formulation of inferences. In creating the tasks, we are based on the belief that selected elements form the basis of propositional logic and are most often used in logical thinking in science education and everyday life.

The survey was performed online using Google Forms due to COVID restrictions valid during the data collection period. Most of the tasks were multiple-choice with one correct answer, except for one biology task in group C. The statistical analysis of the collected data was realized in IBM SPSS version 27.

4 Results

Based on the answers to the 15 tasks, we evaluated the students' results. Each correct response was evaluated 1 point and 0 for incorrect or missing responses. The total test score available was 15 points. The statistical analyses were performed to examine the students` overall test scores and success rate in the three task-groups, including elements of propositional logic. The test scores were compared according to the respondent`s study programme, type of secondary school, gender, and age. Statistical analyses were also performed to determine the success rate in solving individual tasks depending on the subject-specific context.

Examining the test results, we found that the variable *score* is not normally distributed according to the Shapiro-Wilk test (df = 497, W = 0.979, p < 0.001). The variable had a negatively skewed distribution (-0.243). Since the distribution of the score variable is neither normal nor even symmetric, we decided to perform nonparametric tests for other analyzes and comparisons, even if the sample is sufficiently large.
The average test score for N = 497 participants was 8.56 (SD = 2.61). The overall correct answer rate was 57.1%, while the range of correct answer rates for all participants was between 6.7% to 100%. The median value was 9 points.

By examining the test results in three task-groups (Table 2), we found that the students were most successful in solving tasks on the formulation of inferences (group C), with a correct answer rate of 69.4%. In task-group A, dealing with quantifiers, students achieved an average performance of 68.0%. The lowest results, 33.8%, were evaluated in tasks with the negation of statements included in group B.

Table 2: Mean score and SD for each task-group

N = 497	Group A	Group B	Group C	Score
Mean	3.40	1.69	3.47	8.56
SD	1.121	1.155	1.282	2.614

Performing the appropriate statistical test, we found significant differences (Friedman Q = 480.2, df =2, p < 0.001) comparing the test results in three task-groups, A, B, and C. Lowest average score was achieved in task-group B on statement negation. Post-hoc analysis shows the different pairwise distributions for groups A-B and B-C, while the test for samples A-C shows that the distributions are the same.

Based on the answers, students consider the tasks on statements negation to be the most difficult, with the other two task-groups having practically the same difficulty level. The students' success rate of group B tasks are remarkably lower than for tasks A and C.

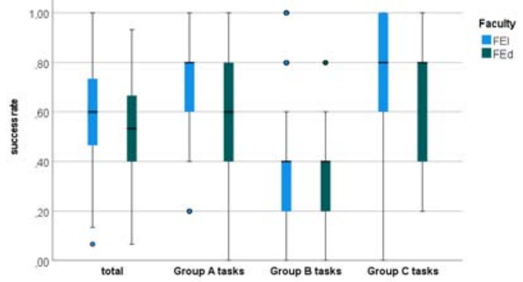
We examined whether there is a difference in the scores achieved in the task-groups according to the students` study program (Q1). For comparison, students were classified into the two faculties based on their study program. We found 63.0% (313) of the respondents studied at FEI and 37.0% (184) at FEd.

Table 3: Basic statistical indicators by faculties

	Group A	Group B	Group C	Score
Mean/FEI	3.50	1.74	3.54	8.78
Mean/FEd	3.24	1.61	3.34	8.20
U	25284.5	28308	25720	25197.5
Sig.	0.019	0.742	0.041	0.019

We see that the average score is higher in all task-groups among FEI students than in FEd (Table 3). Based on the Mann-Whitney test, there is a significant difference (U = 25197.5, p = 0.019) between the average scores of the students studying at FEI and FEd in average total scores and task-groups A and C.

Figure 1: Score distribution and comparison of the test performance for each task-group in terms of respondents’ study programme by faculty



In the next step, we examined the results according to the context of the tasks (Q2). Within each task-group, we compared the results of the respondents of each subject-related task in terms of study programs.

Table 4: Descriptive statistical indicators by task context

	Math	EL	Physics	Bio	Chem
N	491	493	491	445	475
Mean	1.76	2.13	1.79	1.95	1.09
Mean/FEI	1.89	2.17	1.88	1.92	1.14
Mean/FEd	1.56	2.08	1.64	1.99	1.02
U	23491	26025	24053	22445	24870
Sig.	0.001	0.052	0.004	0.426	0.208

Considering the task context, we found significant differences (Friedman Q = 346, df = 4, p < 0.001), and the best results were achieved in real-life tasks (2.13 points). Post-hoc analysis shows that the distributions are the same for Math-Physics, Math-Bio, and Physics-Bio samples. All other samples have different pairwise distributions. That is, the distribution of the points of tasks with context Math-Physics-Bio is approximately the same, except that in the part of Everyday life (EL), points are higher, and in the Chemistry part, points are lower.

If the averages are compared according to the faculties (Mean/FEI and Mean/FEd), there is a significant difference only for mathematics and physics tasks, where the mean score is significantly higher for FEI students (Table 4).

We examined whether there is a difference in students' performance of task-group solutions in terms of gender and age (Q3). The scores obtained in the knowledge test were compared according to the gender and age of the respondents with a Mann-

Whitney U-test. By age, 401 students (81.0%) were between 18 and 25 years old, and 94 students (19.0%) were over 25 years old.

Table 5: Descriptive statistical indicators by age

	Group A	Group B	Group C	Score
Mean/18-25	3.39	1.70	3.45	8.54
Mean/ 25+	3.46	1.68	3.56	8.70
U	18083.5	18228	18201	18409.5
Sig.	0.527	0.605	0.594	0.724

The analysis results show no significant differences in average performance (Mann-Whitney U =18409.5, p = 0.724) in students' age compared to groups 18-25 years and over 25 years (Table 5).

Among the respondents, 310 (62.6%) were women, and 185 (37.4%) were men. However, according to gender, there is no significant difference only in task-group A, in the other task-groups, and in the actual test, we found significant differences between the male and female respondents (Table 6).

Table 6: Descriptive statistical indicators by sex

	Group A	Group B	Group C	Score
Mean/female	3.36	1.58	3.39	8.34
Mean/male	3.46	1.89	3.61	8.96
U	27349.5	25150	25584	24718
Sig.	0.373	0.017	0.039	0.010

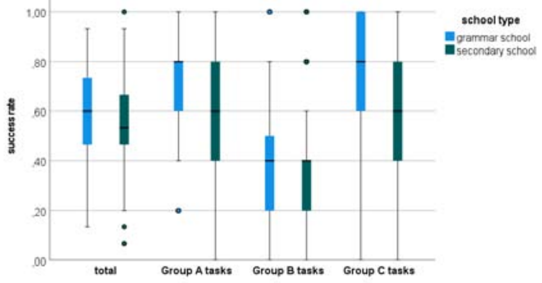
The following analysis concerned the difference in the test results according to the type of secondary school the students finished (Q4). A total of 199 (40.1%) students participating in our survey graduated from secondary grammar schools (SGS), and 297 students (59.9%) completed other secondary schools (SS), which are also schools providing vocational training ending with a graduation certificate. We examined students' success rate in task-groups regarding secondary school finished.

Table 7: Descriptive statistical indicators by secondary school type and test results

N = 496	Group A	Group B	Group C	Score
Mean/SGS	3.53	1.84	3.65	9.02
Mean/SS	3.31	1.60	3.34	8.25
U	26564	26518.5	25387.5	24716.5
Sig.	0.048	0.043	0.006	0.002

As we observed, the mean score was higher for students who attended secondary grammar school in all task-groups (Table 7). Comparing these scores with the Mann-Whitney test, we found that students who finished secondary grammar school achieved significantly better results (U = 24716.5, p = 0.002) in the average total score and three task-groups.

Figure 2: Score distribution and comparison of the test performance for each task-group in terms of respondents' type of secondary school education.



We assume that the test results of secondary grammar school graduates are independent of the study program in which they are currently studying. The higher level of education and content structure of the grammar school education provides a suitable basis for the students to pass the logic test with better results than the other students. We can show the independence of the test results by examining only the secondary grammar school graduates according to the students' faculty. Of the respondents, 199 graduated from grammar school, 110 studied at FEI, and 89 studied at FEd.

Table 8: Descriptive statistical indicators by faculty for students who graduated from secondary grammar schools

N = 199	Group A	Group B	Group C	Score
Mean/FEI	3.58	1.97	3.70	9.25
Mean/FEd	3.46	1.69	3.58	8.73
U	4663.5	4461	4560	4274.5
Sig.	0.553	0.265	0.390	0.122

The Mann-Whitney test results showed no significant differences in average performance (U = 4274.5, p = 0.122) between the students studying at FEI and FEd (Table 8). So grammar school graduates passed the test equally regardless of the study program. However, according to the result obtained for question Q1, we found a difference in the result of the students of the two faculties in the whole sample. This result means that the difference is caused by the respondents who attended non-grammar schools. In Table 7, we see that the SGS students' overall score is 9.02, which is significantly higher than the score of other secondary school graduates, which is 8.25 points on average.

If we examine the results of non-grammar school students, we see that the scores caused the difference between the results of the two faculties.

Table 9: Descriptive statistical indicators by faculty for students who graduated from secondary schools

N = 297	Group A	Group B	Group C	Score
Mean/FEI	3.45	1.62	3.46	8.52
Mean/FEd	3.01	1.56	3.10	7.67
U	7480	9362.5	7785	7758.5
Sig.	0.002	0.786	0.009	0.009

The statistical test showed no significant difference in their results only in task-group B (p = 0.786).

5 Conclusions

The results justify our assumption that FEI students achieve better results in solving the tasks compared to the performance of FEd students. This result can be explained by the fact that the study programmes of the two faculties have different curricula. While the FEd focuses primarily on the general professional preparation of students and the acquisition of pedagogical and methodological competencies, the education of FEI students focuses on subjects used in economics and business that require strategic and logical thinking. Our conclusion shows that students' chosen orientation in higher education is typically related to the successful application of formal logic and its subject and everyday contexts.

The results show significant differences in respondents' success depending on the task's subject-specific context. The respondents were least successful in areas of propositional logic in tasks with a context from chemistry. This result can be explained by the fact that chemistry is abstract and unpopular among students. In the case of chemical tasks, we also found that the presence of chemical concepts made them completely uncertain in the solution of the task, despite the fact that the solution did not require special chemical knowledge. At the same time, the faculty of teacher education study program include the chemistry teacher program, for which the number of students applying is deficient, so their knowledge is not significantly reflected in our results.

We could observe that tasks with statement negation achieved the lowest success rate, and these tasks were also considered the most difficult by the students. The anomalies in the language explain the uncertainty that appeared to a greater extent in the tasks where the statements had to be negated. For example, an anomaly that often appears in ordinary language: the negation of "everything" is "nothing" that does not follow the rules of formal logic, which Csapó (2018) and Chomsky (2006) have pointed out.

Our gender assumptions have not been substantiated. We found significant differences between the male and female respondents. We believe several factors influence the gender gap, so we cannot draw far-reaching conclusions. Our assumption for this age group has been confirmed. Both faculties offer correspondence training, in which the older generation also participates. The fact that older students have more life experience does not mean that their post-formal cognitive development is at a higher level in formal logic than younger students. As we observed, the mean score was higher for students who attended secondary grammar school in all task-groups. Grammar school education still provides better preparation in the knowledge of formal logic.

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Primary Paper Section: A

Secondary Paper Section: AM