OPTIMIZATION OF THE COMPLEX MULTI-FACILITY LOCATION PROBLEM USING MICROSOFT EXCEL

[a]PETR NĚMEC, [b]MICHAL ŠMEREK, [c]ONDŘEJ PEKAŘ, [d]MIROSLAV PECINA

University of Defence, Kounicova 65, Brno 662 10, Czech Republic
email: [a]german.petr@seznam.cz, [b]michal.smerek@unob.cz, [c]ondrej.pekar@unob.cz, [d]miroslav.pecina@unob.cz,

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Abstract: The article presents the possibility of solving a complex Multi Facility Location Problem in Excel and its use in military practice. Based on a simple model example (25 branches, 5 centers), it gradually verifies the individual steps leading to the solution of the optimization requirement of increasing (decreasing) the number of centers, weight of branches, capacity of centers, and forbidden (permitted) areas. These examples are solved using the evolutionary method in Excel. Finally, this method solves an example in the range of 100 to 3 and the result is verified by calculation by the metaheuristic method. This demonstrates the ability to solve this type of tasks in the proposed way.

Keywords: Multi-Facility Location Problem (MFLP), Excel; Solver; Evolutionary algorithm; Simulated Annealing; logistics; benchmark instance

1 Introduction

In our work (1), we have shown that MFLP tasks can be solved using office software on personal computers. In the work (2) the method of solution and test tasks were presented, on which the solution of MFLP with weight was verified. To get closer to solving practical problems, we tried to implement other options into the method. Thus, we came to the proposal of solving complex tasks, including the weight of points - MFLP-W, the capacity of centers - MFLP-C and work with forbidden or, conversely, ordered areas - MFLP-A of the respective MFLP-R. Areas are defined as convex polygons.

The purpose of this work is to demonstrate the capability of Excel-Solver's evolutionary procedure to solve the MFLP. The test task is designed so that partial evaluation of the obtained results is possible without the need for additional control calculations. It has been confirmed that the proposed methodology can successfully address intricate test cases, regardless of the quantity of points, centers, and areas involved, provided that simple test cases can be tackled using this approach. In (1) and (2), the dependence of the complexity of the test task and hardware on the achieved results was investigated. It was found that the inaccuracy of the calculations using the evolutionary method can be successfully compensated by repeating the calculations.

The article is arranged as follows. After reviewing the literature on this topic in section 1.1. and its possible use on the example of military practice in section 1.2., Section 1.3 outlines the presentation of the objective of this study, as well as the formulation of the MFLP-W, MFLP-C, MFLP-A, and MFLP-R problems. Section 2. describes the methods used, the comparative task and the procedure leading to the solution of the required type of MFLP. The test task was designed based on the results of works (1) and (2). Calculations are performed on a regular PC. The test task is mainly used to verify that the design of the problem-solving procedure is correct, the accuracy of the result in this phase of verification is not emphasized. The results of the optimization of individual tasks and their combinations are presented in section 3. There are also listed the problems that were observed for individual tasks and the resulting limitations. Section 4 presents a discourse and draws conclusions for this manuscript.

1.1 Related work

In the literature review, we focused on solving the above types of problems - using heuristic methods and the using of Excel in solving localization problems.

The paper *An efficient algorithm for facility location in the presence of forbidden regions* (3) examines a constrained Weber problem. This work addresses the problem of locating a new facility in convex polygonal restricted regions. Minimize the weighted distance between the new facility and n pre-existing facilities. A forbidden region is assumed to be a plane's restricted travel and facilities area. It is also accepted that this setting uses the Euclidean-distance metric to measure distance. This work presents a nonconvex programming approach. Iteratively solving unconstrained problems yields a local optimum for the starting restricted problem. The material offers numerical examples.

The Weber problem involves placing a new facility on a two-dimensional plane among a finite number of pre-existing facilities. Minimize the weighted sum of distances between the new and existing facilities. This issue is changed when facilities are on opposite sides of a linear obstacle. Rivers, highways, borders, and mountains are frequent barriers. Manuscript *Planar Weber location problems with line barriers* (4) discusses non-convex optimization techniques and structural insights. Distance function and barrier passage number and placement determine the difficulty.

Designing a distribution network in a supply chain system: Formulation and efficient solution procedure (5) examines the process of establishing an effective distribution network. The essay addresses supply chain distribution network architecture. This comprises finding appropriate locations for production plants and distribution warehouses and developing successful techniques for getting items from the plants to the warehouses and from the warehouses to end customers. The study addresses these issues. The goal is to find the best plant and warehouse numbers, locations, and capacities to meet customer demand while minimizing distribution network costs. Studying reveals storage and plant capacities. This computational study evaluates production and distribution planning integration. A mixed integer programming model and efficient heuristic solution were developed for the supply chain system problem.

An efficient solution method for Weber problems with barriers based on genetic algorithms (6) determines the optimal location of a new facility in relation to a pre-existing set of facilities in a two-dimensional plane, accounting for convex polyhedral barriers. Barriers are assumed to be areas where facilities and travel are prohibited. A few convex subproblems can simplify the non-convex optimization issue. The Weiszfeld algorithm solves these subproblems, especially those using the Weber objective function and Euclidean distances. The present work uses a genetic algorithm to iteratively choose subproblems to quickly find a solution for the overarching problem. Visibility arguments reduce the number of subproblems to consider, and numerical examples are provided.

Placing a finite size facility with a center objective on a rectangular plane with barriers (7) addresses finite-size 1-center placement on a rectangular plane with barriers. As described in reference (7), the proposed solution places a finite facility with a center objective on said plane. Barriers impede amenities and movement. Facilities are located in cells. Cells are evaluated by their corners to meet the center's minimax target. The facility dominated when completely enclosed in cells with one, two, or three corners. When the facility intersects gridlines, distance function analysis is difficult. This case's difficulties have been studied and translated into a linear or nonlinear program, depending on the viable region's convexity. A numerical example illustrates the paper's complexity study.

Integrated use of fuzzy c-means and convex programming for capacitated multi-facility location problem (8) solves a capacitated multi-facility location problem using a fuzzy c-means clustering approach. The challenge involves capacitated supply centers serving known demand sites. Fuzzy c-means and convex programming are used. The fuzzy c-means algorithm assigns data points to several clusters with different membership levels. This feature distributes demand among supply centers. An gradual strategy starts with two and ends when each cluster has enough capacity to match its demand. Each cluster group and model is treated as a facility location challenge. Next, convex programming optimizes transportation costs to refine each fuzzy c-means facility location problem. The proposed method has been tested in several facility location scenarios and compared to center of gravity and particle swarm optimization algorithms. This study presents real-world asphalt producer statistics from Turkey. The numerical results show that the suggested methodology beats traditional fuzzy c-means and the combined use of center of gravity strategies for transportation costs.

Two meta-heuristics for a multi-period minisum location-relocation problem with line restriction (9) investigates a multi-period minisum location-relocation problem with rectilinear distance. The mixed-integer nonlinear programming (MINLP) paradigm includes a line-shaped barrier limitation. The barrier starts uniformly in the aircraft. The model minimizes the total costs related to the predicted weighted barrier distance between the new facility and the current ones and relocation costs that rely on location within the planning horizon. Next, a restricted area-based lower limit is set. Numerical examples validate the model. Results show that the optimization software can solve smaller difficulties. The optimization software can't solve large issues quickly. Genetic and imperialist competitive meta-heuristics are proposed in the paper. Finally, the results are compared.

Article *Combining possibilistic linear programming and fuzzy AHP for solving the multi-objective capacitated multi-facility location problem* (10) proposes a solution. The technique evaluates quantitative and qualitative aspects from many angles. Decision-makers must consider both factors to accurately represent complex real-world applications. This study presents a novel approach that integrates a two-phase possibilistic linear programming technique and a fuzzy analytical hierarchical process method to optimize two objective functions, "minimum cost" and "maximum qualitative factors benefit," in a four-stage supply chain network involving suppliers, plants, distribution centers, and customers, while accounting for vagueness. This study's numerical example shows the methodology's findings. Conclusions discuss this approach's benefits.

INSPM: An interactive evolutionary multi-objective algorithm with preference model (11) models DM preferences using the Interactive Non-dominated Sorting algorithm with Preference Model (INSPM). The IN-SPM identifies a non-uniform sampling method for the Pareto-optimal front that uses detailed sampling for the decision maker's favorite regions and coarse sample for the non-preferred regions. A Radial Basis Function (RBF) network is used to calculate the Decision Maker (DM) utility function from ordinal data from DM queries. INSPM invokes the DM's preference model using DCD density control. This strategy improves Pareto-optimal front sampling density by increasing sampling in favored regions and decreasing it in non-preferred regions.

In *The capacitated multi-facility weber problem with polyhedral barriers: Efficient heuristic methods* (12), polyhedral barriers prevent facility placement and transit. Transportation costs depend on both linear distances and polyhedral barrier sizes and location. This circumstance creates a non-convex optimization problem, making solution difficult. This paper proposes location-allocation and discrete approximation heuristics tailored to the problem. Randomly generated test instances underwent extensive computational experiments. The results show that the heuristic approaches are successful and yield promising results for this difficult topic.

Exact and approximate heuristics for the rectilinear Weber location problem with a line barrier (13) expands the multi-Weber facility location problem to include rectilinear-distance and crossings through non-horizontal line barriers. For the single-facility situation, a divide-and-conquer precise heuristic outperforms other literature heuristics. An alternate-location-allocation heuristic using precise and inexact methods solves the problem of managing multiple facilities. Large instances can be handled with a polynomial-time heuristic. This method produces near-optimal, fast solutions with a small gap. A benchmark converts the core problem into a p-median problem for testing. Experimental results show that the recommended heuristics work. These heuristics produce high-quality results in reasonable time.

This paper addresses the capacitated multi-facility Weber issue using rectilinear, Euclidean, squared Euclidean, and lp distances. Identifying m Euclidean plane facilities with limited capacity to satisfy n clients while minimizing transportation costs is the issue. The distance between clients and facilities and the number of units being delivered determine the cost of transportation, which is publicly available. This work proposes three new heuristic methods based on simulated annealing, threshold acceptance, and genetic algorithms. Benchmark results show that heuristics create high-quality solutions. The paper *Solving the Capacitated Multi-Facility Weber Problem by Simulated Annealing, Threshold Accepting and Genetic Algorithms* (14) found that the heuristic using simulated annealing and the two-variable exchange neighborhood structure performed best (14).

Facility placement on a two-dimensional plane is restricted in restricted planar location challenges. Congested polygons are highly inhabited. Due to restricted space, these places make facility location difficult, however transportation is achievable at a fixed cost. This study shows that determining optimal locations in congested regions on the Euclidean plane is a broader version of the two extensively researched restricted planar facility location problems involving forbidden regions and barriers. The constrained planar location problem is addressed by three metaheuristic algorithms with local search procedures. User interface modules execute algorithms on test instances and do computational experiments. The study shows that the proposed algorithms can solve large issues (15).

In addition to our own resources (1) and (2), the MFLP solution with weight, capacity, and areas in Excel, or other widely available software, is not described in the available literature. Because the terminology is ambiguous, Section 1.2. defines the problems and their designations used in this work.

1.2 Application of MFLP in supporting the decision-making process in planning the deployment of critical artillery support locations

Solving sophisticated MFLP tasks using MS Excel can find applications in several fields. The previous article outlined some possible applications of the MFLP method (1). These were applications for military logistics and support of the Tactical Decision Support System. The use of the MFLP method is shown in this article in the practical example of logistical support of artillery units in combat.

Artillery has played and continues to play an essential role on the battlefield, as evidenced by current conflicts, especially the one in Ukraine (16,17). However, the activity of artillery units is tied to logistics capabilities. This is mainly in the material supply, without which artillery cannot provide fire support. Artillery significantly burdens logistical support and thus plays a critical role in providing continuous fire support on the battlefield (18).

Artillery fire units have a limited amount of supplies available for the execution of firing missions, which they are able to carry themselves. This is particularly the amount of ammunition, fuel, and other materials. These supplies must be replenished during combat (19).

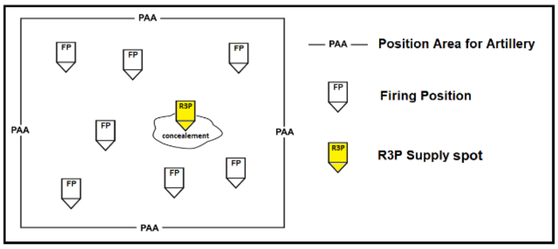

Figure 1. Scheme of artillery units in combat

Artillery units are within the operation deployed in areas, so-called Position Area for Artillery, from where they provide fire support and in which they can maneuver without any restrictions. Inside the Position Area for Artillery, it is possible to distinguish (among other things) two basic types of positions. Firing positions are specific locations where artillery units conduct fires. The number of firing positions depends on factors like tactical situation, number of units, and so on (20,21).

The second type of position is designated to hide artillery units and replenish supplies. After completing the firing mission, the artillery units maneuver to a concealed position, remaining in the breaks between firing missions. These concealments are chosen to protect from direct observation from the enemy while being close to the firing positions. In these locations, material maintenance is carried out in combat conditions, but in particular, supply spots are established, where ammunition, fuel, and other materials are replenished. In a military environment, these resupply spots are called R3P - rearm, refuel, resupply points. R3P spots are established in advance (if the tactical situation allows it) close to firing positions so that the resupply takes place as efficiently as possible. The amount of these supply spots in the area depends on the number of firing positions. However, it is generally possible to say that one supply spot can provide supplies for multiple units – thus also firing positions in the area (22). For the article, we can thus distinguish two basic types of locations. Firing positions represent branches, while supply spots (R3P points) represent the centers we try to find using the specified criteria.

Using the MFLP method to objectively find locations for these supply spots based on the location of firing positions is possible. Such an application can then serve the needs of the commander's decision on the location of these supply spots in the Military Decision Making Process (MDMP) process and thus facilitate his decision. The number and locations of firing positions and the corresponding supply spot(s) are chosen based on several criteria. These criteria can be the tactical situation, the operation phase, the terrain conditions, and the possibilities of concealment or the number of units operating. Artillery units can use firing positions depending on the nature of the firing missions. For this, the weighting criteria of the MFLP algorithm can be used. Individual firing positions can be prioritized by weights, depending on the tactical situation or, e.g., the operation phase. In determining the positions of supply spots under these conditions, it is possible to achieve that they will be chosen concerning the weight - the priority of the firing positions. We can also include the capacities of individual supply spots in the algorithm. The capacity of supply spots expresses the ability of the supply spot to provide ammunition and other material for only a certain number of spots, i.e., in our case, the units in the firing position. The number of supply spots (centers) can thus be adjusted to ensure sufficient supplies for all units in firing positions in the position area for artillery. The least but not last criteria with which the MFLP algorithm can work are area constraints. When entering the calculation of supply spots, it is possible to determine the areas in which they must or must not be located. This means it is possible to include the boundaries designated for artillery units in calculating supply spot locations so that these spots are located inside this area. Similarly, it is then possible to proceed with the condition that the supply spots are not located inside the forbidden area. Forbidden areas may represent places where the geographical conditions do not allow for the deployment of supply spots, etc.

Position determination of the supply spots need not pose a problem when it is necessary to determine the supply spot (center) for only a few firing positions (spots). However, if there are more spots, an objective assessment of the location of the supply spots (centers) can be problematic.

1.3. Problem formulation

The formulation of the problem is processed for each test separately. Tests are designed from the simplest to the most complex.

1.3.1 TEST A-1 (5/25)

There exist a set of *p* points, for which the coordinates of their respective positions are known. There are *c* centers where the coordinates of their positions are unknown. The variable *c* is a positive integer and *p* is an integer greater than or equal to *c*. The aim is to determine the locations of the centers in such a way that the total distance between each center and its designated points is minimized. Each individual point will be allocated to a single center.

A practical example is an artillery battery with existing firing positions (*Pj*) for which needs to be set up supply spots (*Ci*) that will provide supplies for artillery units in firing positions. The firing positions' coordinates are established, while the supply locations will be formed anew. The specific locations of the supply spots and the corresponding firing positions they are intended to serve have not been specified. For this test is *c*=5 and *p*=25.

The problem can be expressed mathematically in the following manner. Let $P_j, j = 1, 2, \dots, p$ be a set of points $P = \{P_1, P_2, \dots, P_p\}$; the coordinates of these points are defined as $[Px_j, Py_j], j = 1, 2, \dots, p$. Let $C_i, i = 1, 2, \dots, c$ be a set of centers $C = \{C_1, C_2, \dots, C_c\}$; The precise coordinates of said points remain unknown, as they are actually under optimization.

The Euclidean distance between a center $C_i = [Cx_i, Cy_i]$, $i = 1, \dots, c$ and point $P_j = [Px_j, Py_j]$, $j = 1, 2, \dots, p$ The value is determined based on the computation outlined in equation [1]. In a general sense, it is possible to utilize any spatial configuration, dimensional quantity, and metric for measuring distance.

$$d_{ij} = d(C_i, P_j) = \sqrt{\left(P_{x_j} - C_{x_i}\right)^2 + \left(P_{y_j} - C_{y_i}\right)^2}, \quad i = 1, \dots, c, \; j = 1, \dots, p \qquad [1]$$

where $d_{ij} = d(C_i, P_j)$ is the distance between center C_i and point P_j; C_{x_i}, C_{y_i} are coordinates of the center C_i, and P_{x_j}, P_{y_j} are coordinates of point P_j.

The solution is therefore the matrix *X* (assignment matrix) described in formula [2]

$$X = (x_{ij})_{c \times p} = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{c1} & \cdots & x_{cp} \end{pmatrix}, where \quad x_{ij} = \begin{cases} 1 \dots assigned \\ 0 \dots unassigned \end{cases} \qquad [2]$$

Each point is assigned to exactly one center. Therefore, in each column there is exactly one "1" and $(c-1)$ "0", so it must be:

$$P_j: \sum_{i=1}^{c} x_{ij} = 1, \qquad j = 1, 2, \dots, p. \qquad [3]$$

Formula [4] displays the optimization function for the MFLP problem. The aforementioned statement pertains to the total of the distances that exist between each point and the centers that have been assigned to them. The objective is to determine the optimal positions of *c* centers in order to minimize the objective function *D*. In the context of two-dimensional space, the number of continuous optimization variables is 2*c*.

$$D = \sum_{j=1}^{p}\left(\sum_{i=1}^{c} x_{ij} d_{ij}\right) \longrightarrow min \qquad [4]$$

1.3.2 TEST A-2 (3/25)

The formulation is the same as for test A-1 (5/25) but *c*=3

1.3.3 TEST A-3 (3/25 W)

The dataset consists of a set of *p* points, each with known positional coordinates and an associated weight. There are *c* centers where the coordinates of their positions are unknown. The variable *c* is a positive integer and *p* is an integer greater than or equal to *c*. The aim is to determine the locations of the centers such that the total weighted distance between each center and its designated points is minimized. Each data point will be allocated to a single centroid that is in closest proximity to it. A practical example is an artillery battery with existing firing positions (*Pj*) for which needs to be set up supply spots (*Ci*) that will provide supplies for artillery units in firing positions. The locations of the firing positions are known; the supply spots will be built as new. In this case the firing positions have a different supply requirements depending e.g. on the phase of operation or may be prioritized for another reason. These different supply requirements are taken into account in the calculation by means of weights. For this test is *c*=3 and *p*=25.

The mathematical expression of this particular problem is identical to that of the instance A-1, but every *Pj* have a weight $w_j, j = 1,2, \dots, p$; where w_j represents a preference for some points over others.

Formula [1] is utilized to calculate the Euclidean distance between a center and any given point. However, the coordinates of the centers are determined by means of formula [5].

For a given solution given by the matrix $X = (x_{ij})_{c\times p}$ the coordinates of all centers can be determined $C_i = [Cx_i, Cy_i]$, $i = 1, 2, \dots, c$ as follows:

$$Cx_i = \frac{\sum_{j=1}^{p} Px_j . w_j . x_{ij}}{\sum_{j=1}^{p} w_j x_{ij}}, \qquad Cy_i = \frac{\sum_{j=1}^{p} Py_j . w_j \cdot x_{ij}}{\sum_{j=1}^{p} w_j \cdot x_{ij}}, i = 1, 2, \dots, c. \qquad [5]$$

Formula [6] demonstrates the optimization of the MFLP-W problem. The aforementioned pertains to the summation of distances that have been weighted between each point and their respective designated centers. The purpose of the function is to calculate the sum of distances between all points and their respective assigned centers, with each distance being weighted.

$$D = \sum_{j=1}^{p}\left(w_j \sum_{i=1}^{c} x_{ij} d_{ij}\right) \longrightarrow min \qquad [6]$$

1.3.4 TEST A-4 (3/25 C)

There exist a set of *p* points, for which the coordinates of their respective positions are known. There are *c* centers where the coordinates of their positions are unknown. The variable *c* is a positive integer, where $c \geq 1$, and the variable *p* is a positive integer, where $p \geq c$. The aim is to determine the locations of the centers in such a way that the total distance between each center and its designated points is minimized. Each individual point will be allocated to a single center. Assign a capacity to each center that restricts the upper limit of points that can be assigned to given center.

A practical example is an artillery battery with existing firing positions (*Pj*) for which needs to be set up supply spots (*Ci*) that will provide supplies for artillery units in firing positions. The firing positions' coordinates are established, while the supply locations will be set up as novel deployments. The specific locations of the supply spots and the corresponding firing positions they are intended to serve have not been specified. Each supply spot shall have a capacity representing the maximum number of points it is able to provide supplies for. For this test is *c*=3 and *p*=25.

The mathematical formulation of this problem is identical to that of example A-1, with the exception that each center possesses a capacity $k_i, i = 1,2, \dots, c$; where k_i represents the maximum number of points that are allocated to a given center.

Any number of points can be assigned to center C_i, but that the capacity of the center must not be exceeded. The relation defined by formula [7] therefore applies.

$$C_i: \ \sum_{j=1}^{p} x_{ij} \leq k_i, \quad i = 1, 2, \dots, c. \qquad [7]$$

Formula [1] is utilized to compute the Euclidean distance between a center and any point in a two-dimensional space. Formula [4] displays the optimization function for the MFLP-C problem. The aforementioned pertains to the aggregate of distances linking each point to its designated centers. The objective is to determine the optimal positions of *c* centers such that the objective function *D* is minimized while ensuring that the capacity of each center is not surpassed. In the context of two-dimensional space, the number of continuous optimization variables is 2*c*.

1.3.5 TEST A-5 (3/25 CW)

The given dataset comprises of *p* discrete points, each with known positional coordinates, a corresponding weight. Every center has a specified capacity. The aim is to determine the optimal locations of the centers such that the total weighted distance between each center and its designated points is minimized, while ensuring that the capacity constraint of each center is not violated.

A practical example is an artillery battery with existing firing positions (*Pj*) for which needs to be set up supply spots (*Ci*) that will provide supplies for artillery units in firing positions. The firing positions' coordinates are established, while the supply locations will be constructed afresh. The specific locations of the supply spots and the corresponding firing positions they are intended to serve have not been specified. Firing posts have a different supply prioritisation represented by weights and each supply spot shall have a capacity representing the maximum number of points it is able to provide supplies for. For this test is *c*=3 and *p*=25.

The mathematical formulation of this problem is the combination of tests A-1 to A-4.

The solution is then the matrix of assignments given in the formula [5], where the center coordinates are calculated as the center of gravity of all assigned points with weights. At the same time, the condition applies that any number of points can be assigned to center C_i, but that the capacity of the center must not be exceeded. This condition is defined by formula [8]:

$$C_i: \ \sum_{j=1}^{p} x_{ij} w_j \leq k_i, \quad i = 1, 2, \dots, c. \qquad [8]$$

Formula [1] is utilized to compute the Euclidean distance between a center and any point in a two-dimensional space. Formula [6] displays the optimization function for the MFLP-CW problem. The aforementioned pertains to the summation of weighted distances between each point and its corresponding assigned centers. The objective is to determine the optimal positions of *c* centers such that the objective function *D* is minimized while ensuring that the capacity of each center is not exceeded. In the context of two-dimensional space, it can be observed that there exist a total of 2*c* continuous optimization variables.

1.3.6 TEST B-1 (3/25 CWA)

The given dataset comprises of *p* distinct points, each having a designated weight and every center with a specific capacity. The coordinates of these points are known. The aim of this study is to determine the optimal positions of centers such that the total weighted distance between each center and its assigned points is minimized, while ensuring that the capacity of each center is not surpassed and that all centers are situated within the specified area.

A practical example is an artillery battery with existing firing positions (*Pj*) for which needs to be set up supply spots (*Ci*) that will provide supplies for artillery units in firing positions. The firing position coordinates are established and must be situated within the designated artillery position area, along with the newly constructed supply spots. The specific locations of the aforementioned supply spots and the corresponding firing positions they are intended to serve have not been delineated. Firing posts have a different supply prioritisation represented by weights and each supply spot shall have a capacity representing the maximum number of points it is able to provide supplies for. For this test is *c*=3 and *p*=25.

The mathematical expression of this particular problem is identical to that of tests A-5, but all centers must be located is designated area.

The areas in which the centers must be located are defined as convex polygons $V_1 V_2 \dots V_v$; with coordinates of vertices $V_k = [Vx_k, Vy_k], k = 1, 2, \dots, v$.

Consider v functions in formula [9] that are linear to x,y:

$$p_k(x, y) = (Vx_{k+1} - Vx_k)y - (Vy_{k+1} - Vy_k)x - (Vx_{k+1} - Vx_k)Vy_k + (Vy_{k+1} - Vy_k)Vx_k \quad [9]$$

$k = 1, 2, \dots, v, \quad v + 1 := 1$

If all v in inequalities in formula [10] are valid, then the center C_i lies in the given area, i.e. in a convex polygon $V_1 V_2 \dots V_v$.

$$p_k(V_{k+2}) \cdot p_k(C_i) > 0 \Leftrightarrow C_i \in Hfp_k, \quad [10]$$

where Hfp_k is a half-plane determined by a straight line $\overleftrightarrow{V_k V_{k+1}}$ and point $V_{k+2}, k = 1, 2, \dots, v, \ v + 1 := 1,$ $v + 2 := 2.$

Formula [1] is utilized to compute the Euclidean distance between a center and any point in a two-dimensional space. Formula [6] depicts the optimization function for the MFLP-CWA problem. Formula [8] demonstrates that it is equivalent to the total of weighted distances between each point and its designated centers. The objective is to minimize the objective function *D* while ensuring that the capacity of the center is not surpassed by determining the optimal locations of *c* centers. In the context of two-dimensional space, it can be observed that there exist a total of 2*c* continuous optimization variables.

1.3.7 TEST B-2 (3/25 CWR)

The dataset comprises a set of *p* points, each with known coordinates, a corresponding weight. Every center has a designated capacity. The aim of the study is to determine the optimal positions of the centers such that the total weighted distance between each center and its assigned points is minimized, while ensuring that the capacity of each center is not surpassed and that all centers are situated outside of the designated restricted area.

A practical example is an artillery battery with existing firing positions (*Pj*) for which needs to be set up supply spots (*Ci*) that will provide supplies for artillery units in firing positions. The firing positions' coordinates are determined, whereas the supply spots are to be constructed anew and ought to be situated beyond the restricted zone. The specific locations of the supply spots and the corresponding firing positions they are intended to serve have not been delineated. Firing posts have a different supply prioritization represented by weights and each supply spot shall have a capacity representing the maximum number of points it is able to provide supplies for. For this test is *c*=3 and *p*=25.

The mathematical representation of this particular problem is identical to that of tests B-1, with the exception that all centers are required to be situated outside of the designated area.

Proceeding again from v functions in formula [9]. If at least one of v inequalities in formula [11] is valid, then the center C_i is not located in the given area, i.e. in a convex polygon [11]

$$V_1 V_2 \dots V_v . p_k(V_{k+2}) \cdot p_k(C_i) < 0 \Leftrightarrow C_i \notin Hfp_k,$$

where Hfp_k is a half-plane determined by a straight line $\overleftrightarrow{V_k V_{k+1}}$ and point $V_{k+2}, k = 1, 2, \dots, v,$ $v + 1 := 1, v + 2 := 2.$

Formula [1] is utilized to compute the Euclidean distance between a center and any point in a two-dimensional space. Formula [6] depicts the optimization function for the MFLP-CWR problem. Formula [8] demonstrates that it is equivalent to the total of weighted distances between each point and its designated centers. The objective is to determine the optimal positions of *c* centers such that the objective function *D* is minimized while ensuring that the capacity of each center is not surpassed. In the context of two-dimensional space, it can be stated that there exist 2*c* optimization variables that are continuous.

1.3.8 TEST B-3 (3/25 CWAR)

The given dataset comprises of *p* points, each with known positional coordinates, a corresponding weight. Every center with a designated capacity. The aim is to determine the locations of the centers such that the total weighted distance between all centers and their designated points is minimized, while ensuring that the capacity of each center is not surpassed. Simultaneously, it is required that one of the centers be situated within polygon A, while two centers must be positioned within polygon B.

A practical example is an artillery battery that can be divided into two platoons, each of which has an assigned area in which it operates. Each platoon operates in its assigned area with existing firing positions (*Pj*) for which needs to be set up supply spots (*Ci*) that will provide supplies for artillery units in firing positions. The firing positions have been identified, while the supply spots are to be constructed anew and must be situated individually for each platoon within their designated region. The precise geographical coordinates of the aforementioned supply spots and the corresponding firing positions they are intended to serve have not been explicitly delineated. Firing posts have a different supply prioritisation represented by weights and each supply spot shall have a capacity representing the maximum number of points it is able to provide supplies for. For this test is *c*=3 and *p*=25.

In this test, a combination of all the previous conditions is applied.

Formula [1] is utilized to determine the Euclidean distance between a center and any point in a two-dimensional space. Formula [6] displays the optimization function for the MFLP-CWAR problem. Formula [8] demonstrates that it is equivalent to the total of weighted distances between each point and its designated centers. The objective is to minimize the objective function *D* while ensuring that the capacity of the center is not surpassed by identifying the optimal locations of *c* centers. In the context of two-dimensional space, the number of continuous optimization variables is 2*c*.

2 Materials and methods

This section describes the hardware used to calculate test tasks, the operating system, and applications. There is also a description of the methods used for optimization and auxiliary methods used in partial calculations.

2.1 Hardware and software configuration

Computational analyses were conducted on two distinct computer systems that possessed varying hardware configurations:

HW 1 - CPU: AMD A10-9620P RADEON R5, 10 COMPUTE CORES 4C + 6G 2.50 GHz. Installed memory 8.00 GB RAM.

HW 2 - CPU: INTEL CORE i7-7700 3.60 GHz. Installed memory 32.00 GB RAM. Software configuration - 64-bit operating system, Windows 10 Enterprise LSTC, MS Excel 2016, 64 bit, part of the Microsoft Office Suite, with the Solver add-in installed.

2.2 Evolutionary method

The evolutionary algorithm is classified as a component of Evolutionary Computations and is categorized among contemporary heuristic-based search techniques. Global optimization is a highly effective problem-solving technique for commonly encountered problems, owing to its adaptable characteristics and strong performance derived from Evolutionary Computation. This technology exhibits efficacy in numerous applications characterized by a high degree of complexity. The Solver add-in in Excel was utilized to compute all the enumerated test tasks.

As per the reference (11), the fundamental characteristics of evolutionary algorithms can be delineated as follows. The initial stage involves the execution of a stochastic sampling procedure. The evolutionary algorithm is designed to sustain a group of potential solutions, known as a population. It is possible that a single individual or a group of individuals with similar objectives may be deemed as the optimal solution, while other members of the population may serve as exemplary models in distinct regions of the exploration space, where a superior resolution can be eventually discovered. The evolutionary algorithm introduces stochastic modifications or mutations to one or multiple individuals within the extant population, thereby generating a fresh prospect solution that may exhibit superior or inferior performance relative to the current population. The evolutionary algorithm endeavors to amalgamate components of extant solutions in order to generate a novel solution that incorporates certain attributes of each "parent". The process of combining elements from pre-existing solutions is achieved through a crossover operation. Ultimately, the evolutionary algorithm executes a selection procedure wherein the individuals deemed the "most suitable" within the population are able to persist, while those deemed the "least suitable" are excluded. In the context of a constrained optimization problem, the concept of "fitness" is contingent upon both the feasibility of the solution, as determined by its adherence to all constraints, and the objective function's value. The process of selection constitutes a crucial stage in the evolutionary algorithm's progression towards increasingly optimal solutions.

The algorithm may be expressed in the form of pseudocode notation.

START

Generate the initial population

Compute fitness

REPEAT

Population

Mutation

Crossover

Selection

Compute fitness

UNTIL population has converged

STOP

This method allows for the configuration of convergence values, mutation frequency, base file size, random number, and maximum time without enhancement. The impact of alterations to these parameters on the computation is unspecified; hence, default values were maintained for all computations. Table 1 presents the values of default parameters.

Table 1. Parameters used for optimization in the Excel-Solver

Parameter	Value
Max time	Unlimited
Iterations	Unlimited
Constraint precisions	0,000001
Convergence	0,0001
Population size	100
Random seed	0
Mutation rate	0,075
Maximum time without improvement	30 sec
Max subproblems	Unlimited
Maximum feasible solutions	Unlimited
Integer optimality	1%

2.3 Metaheuristic method

Previous studies have confirmed the effectiveness and outcomes of the MFLP approach through the utilization of metaheuristic techniques (1,2). The utilization of the metaheuristic approach, which is founded on the Simulated Annealing principle, has been employed to evaluate the efficacy of MS Excel in addressing the challenge of continuous multi-variable optimization. The principle being referred to draws inspiration from the annealing process utilized in metallurgy, which involves subjecting a material to controlled heating and cooling in order to mitigate any defects present. The authors implemented the calculation procedure using the C++ programming language. Further elaboration on the metaheuristic approach and the specific algorithm utilized can be found in prior scholarly investigations (1).

2.4 Benchmark values

Table 2 displays the coordinates of the points P utilized in tests A and B. The point topology utilized in the artificial benchmark A has been intentionally structured in a unique manner, thereby enabling the anticipation or partial anticipation of a solution.

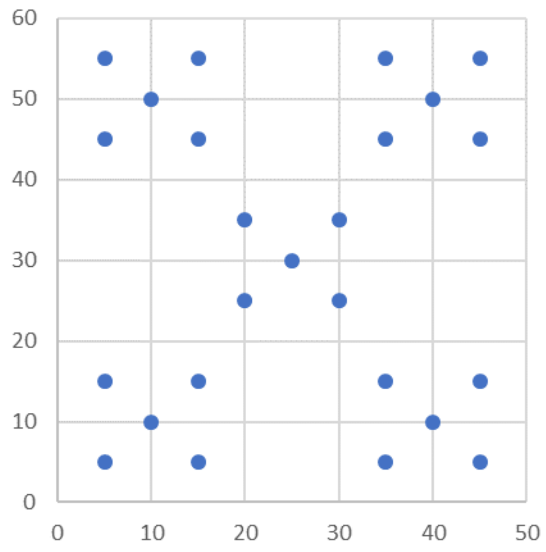

Figure 2. Benchmark A, B

Table 2. Coordinates of points

x	y
5	5
15	5
35	5
45	5
10	10
5	15
15	15
35	15
45	15
40	10
5	45
15	45
35	45
45	45
10	50
5	55
15	55
35	55
45	55
40	50
20	25
30	25
20	35
30	35
25	30

Table 3 lists the terms used and their explanations.

Table 3. Definitions of terms used

Name of term	Explanation
Minimal	Minimal value from set of 100 results
Average	Average value from set of 100 results
Dif. ref-min	Difference between reference and minimal value (%)
Standard dev.	Standard deviation for set of 100 results
Avg. time	Average time of computing one result in set of 100 results

3 Results

The results section provides a concise and objective summary of the experimental and computational data and presents the findings obtained from the conducted research. Data and results are shown in the form of tables and figures, followed by a comprehensive interpretation of the results.

3.1. Results, TEST A

Test tasks A are optimized on HW1, Test tasks A are used to verify the applicability of the method and to confirm the expected result.

3.1.1 TEST A-1 (5/25)

The task of finding 5 centers for 25 points is solved first. The procedure is described in detail in Section IV. The optimization conditions and the result are in Table 4.

Table 4. Results of test A-1

Number of centers		5	
Number of points		25	
Total distance		141.4683926	
Center	**Number of assigned points**	**x**	**y**
1	5	40.00578353	10.00500703
2	5	39.99977012	49.99824379
3	5	9.994063198	50.00695307
4	5	25.00818919	30.01828226
5	5	10.00049883	10.00833797

For this problem, it can be predicted that the correct solution will be the position of the centers in the middle of each group of five points. This prediction is confirmed by calculation and graphically shown in Fig. 3.

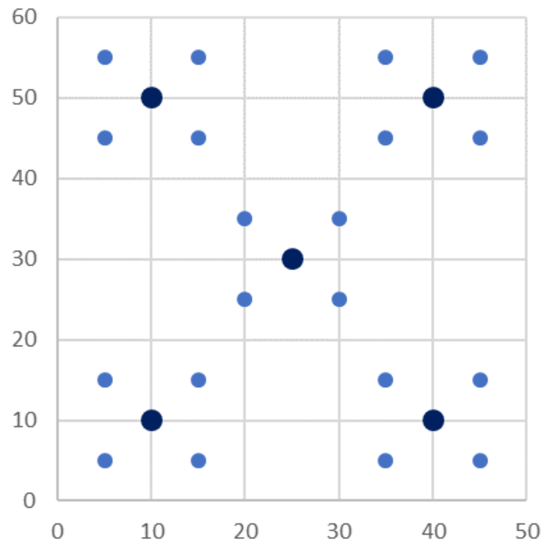

Figure 3. Result of optimization test A-1

3.1.2 TEST A-2 (3/25)

The number of centers is reduced to 3 in this test. Points are assigned to the center using the MATCH and COUNTIF functions. The optimization conditions and the result are in Table 5.

Table 5. Results of test A-2

Number of centers		3	
Number of points		25	
Total distance		294.6004439	
Center	**Number of assigned points**	**x**	**y**
1	13	25.03041478	45.25362532
2	6	10.11654392	10.18144429
5	6	39.98807409	10.01568527

For this problem, it can be predicted that the centers are moved and number of assigned points are changed. This prediction is confirmed by calculation and graphically shown in Fig. 4.

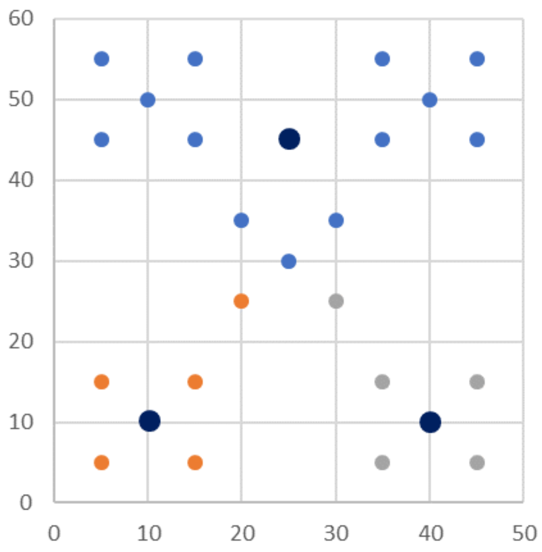

Figure 4. Result of optimization test A-2

3.1.3 C.TEST A-3 (3/25 W)

In this test, the weight is added to three points. The optimization conditions and the result are in Table 6.

Table 6. Results of test A-3

Number of centers		3	
Number of points		25	
Weights		Points [5,5]; [45,5]; [25,30] have a weight of 3, the remaining points have a weight of 1.	
Total distance		341.7222172	
Center	**Number of assigned points**	**x**	**y**
1	5	7.173812421	7.138157584
2	15	25.0017582	38.30947595
3	5	42.63724447	7.365523656

For this task, it can be predicted that the position of the centers will shift towards the points with weight 3 and a change in the number of points assigned to the centers. This prediction is confirmed by calculation and graphically shown in Fig. 5.

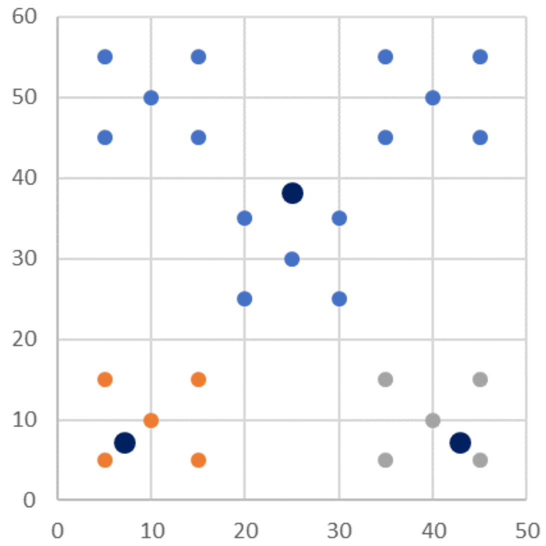

Figure 5. Result of optimization test A-3

3.1.4 TEST A-4 (3/25 C)

Furthermore, the task of finding 3 centers for 25 points is solved, in which the maximum capacity is set for the centers - the capacity corresponds to the number of assigned points. The optimization conditions and the result are in the Table 7.

Table 7. Results of test A-4

Number of centers		3	
Number of points		25	
Capacity of centers		All centers have a maximum allowed capacity (number of assigned points) 9.	
Total distance		331.3074116	
Center	**Number of assigned points**	**x**	**y**
1	8	12.37293583	44.08488544
2	8	35.47962436	42.66568074
3	9	33.91542714	6.917351073

For this task can be predicted, that the position of the centers will shift and the number of points assigned to the centers will change. This prediction is confirmed by calculation and graphically shown in Fig. 6.

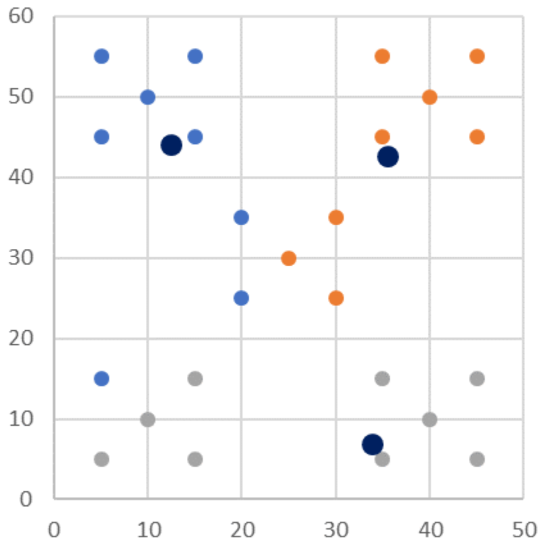

Figure 6. Result of optimization test A-4

3.1.5 TEST A-5 (3/25 CW)

This task combines the capacity of the centers and the weight of the points. The maximum capacity is set for the centers, i.e. the sum of the weights of the assigned points. Weight of the assigned points is the same as in test A-3 (3/25 W). The optimization conditions and the result are in Table 8.

Table 8. Results of test A-5

Number of centers		3	
Number of points		25	
Capacity of centers		All centers have a maximum allowed capacity (sum of weights of assigned points) of 11. Points [5,5]; [45,5]; [25,30] have a weight of 3, the remaining points have a weight of 1.	
Total distance		407.6103343	
Center	**Sum of weights of assigned points**	**x**	**y**
1	11	15.0877231	47.09785915
2	9	41.08188208	15.27571591
3	11	13.55289245	14.13270853

For this task can be predicted, that the position of the centers will shift, and the number of points assigned to the centers will change. This prediction is confirmed by calculation and graphically shown in Fig. 7.

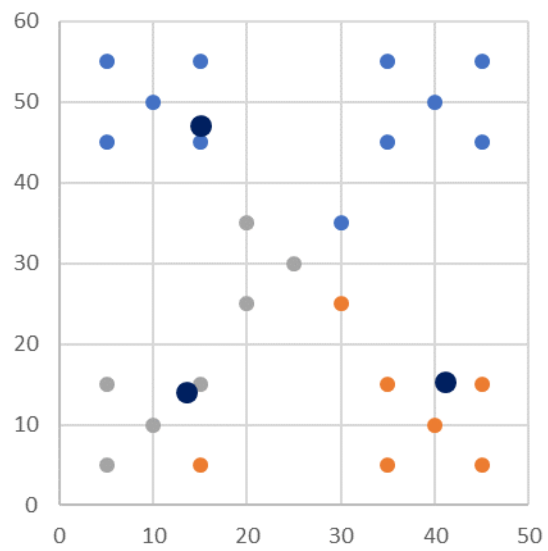

Figure 7. Result of optimization test A-5

3.2 Results, TEST B

Test tasks B are optimized on HW1, Test tasks B are used to verify the applicability of the method and to confirm the expected result.
To optimize the tasks in which the area in which the centers can be located is determined, it is necessary to add to the Excel workbook functions that can determine the mutual position of the center and the convex polygon. The output of these functions is the logical value TRUE if the center lies inside the polygon. By combining these values, different conditions for optimization can then be set. Benchmark A test tasks are used to verify the applicability of the method and to confirm the expected result. Two areas are prepared in the test tasks. The coordinates of the vertices of convex polygons A and B are given in Table 9.

Table 9. Coordinates of convex polygons A and B

Ax	Ay	Bx	By
15	55	5	5
14	54	35	7
21	35	30	25
25	36	6	21
45	52	2	10

3.2.1 TEST B-1 (3/25 CWA)

This task combines center capacity and point weight and adds work with areas. The maximum capacity is set for the centers, i.e. the sum of the weights of the assigned points, all centers must lie in area A or B or both. The optimization conditions and the result are in Table 10.

Table 10. Results of test B-1

Number of centers		3	
Number of points		25	
Capacity of centers		All centers have a maximum allowed capacity (sum of weights of assigned points) of 11.	
Number of centers in areas A,B		3	
Total distance		415.3432088	
Center	**Sum of weights of assigned points**	**x**	**y**
1	11	33.03465031	14.06651739
2	9	9.304041817	15.26529007
3	11	35.00187268	46.77105185

For this task can be predicted that there will be a shift position change centers and the number of points assigned to the centers. This prediction is confirmed by calculation and graphically shown in Fig. 8.

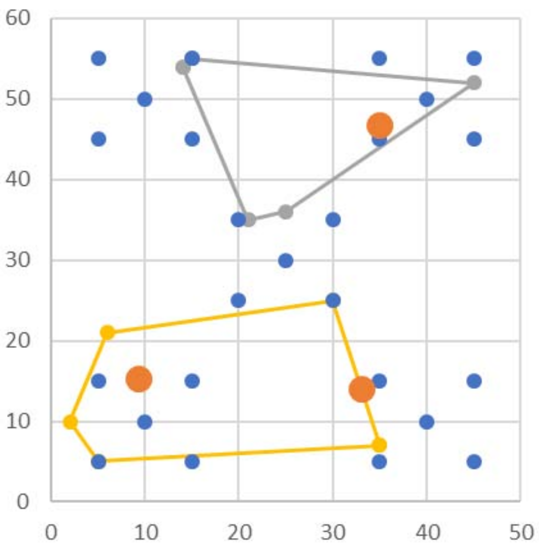

Figure 8. Result of optimization test B-1

3.2.2 TEST B-2 (3/25 CWR)

This task combines center capacity and point weight and adds work with areas. The maximum capacity is set for the centers, i.e. the sum of the weights of the assigned points, all three centers must lie outside areas A and B. The optimization conditions and the result are in Table 11.

Table 11. Results of test B-2

Number of centers		3	
Number of points		25	
Capacity of centers		All centers have a maximum allowed capacity (sum of weights of assigned points) of 11.	
Number of centers in areas A,B		0	
Total distance		426.1972904	
Center	**Sum of weights of assigned points**	**x**	**y**
1	11	24.36114487	29,68105772
2	9	30.17352628	53,51005192
3	11	36.25803366	6,988918277

For this task can be predicted that there will be a shift position change centers and the number of points assigned to the centers. This prediction is confirmed by calculation and graphically shown in Fig. 9.

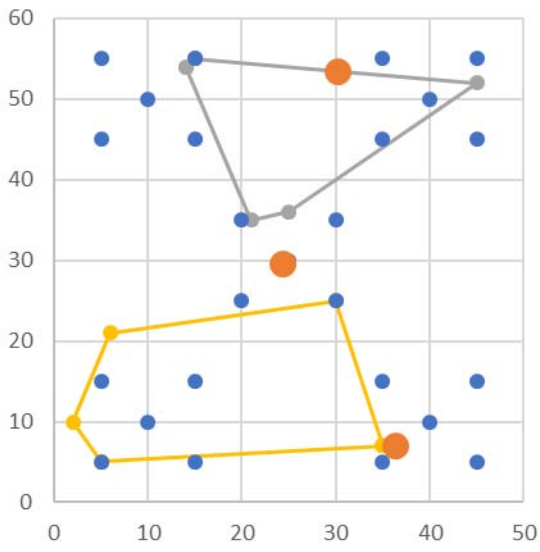

Figure 9. Result of optimization test B-2

3.2.3 TEST B-3 (3/25 CWAR)

This task combines center capacity and point weight and adds work with areas. The maximum capacity is set for the centers, i.e. the sum of the weights of the assigned points, one center must lie into area A and two into area B. The optimization conditions and the result are in Table 12.

Table 12. Results of test B-3

Number of centers		3	
Number of points		25	
Capacity of centers		All centers have a maximum allowed capacity (sum of weights of assigned points) of 11.	
Number of centers in areas A,B		Area A - 1, area B - 2	
Total distance		414.918009	
Center	**Sum of weights of assigned points**	**x**	**y**
1	9	8.738383737	14.06651739
2	11	32.98934049	15.26529007
3	11	35.43413036	46.77105185

For this task can be predicted, that there will be a shift position change centers and the number of points assigned to the centers. This prediction is confirmed by calculation and graphically shown in Fig. 10.

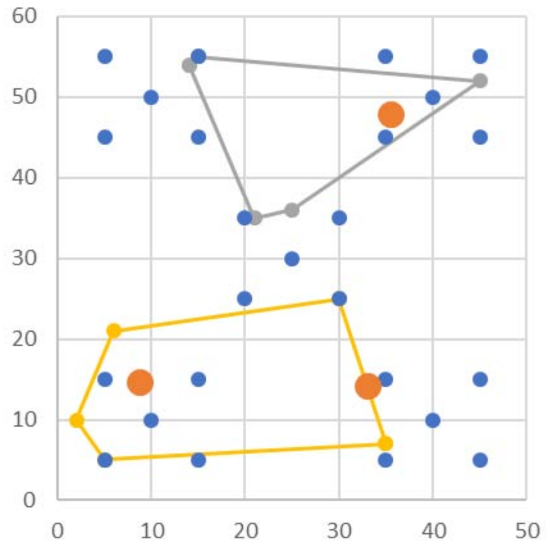

Figure 10. Result of optimization test B-3

4 Discussion & Conclusion

This article follows the original research on the possibilities of solving the MFLP problem using MS Excel software. The capabilities of solving the MFLP tasks using MS Excel were verified by metaheuristic method and published in previous research (1,2) including the influence of HW equipment and the number of repetitions of tasks on the accuracy of the results. The aim of this article was to expand the original research and to demonstrate the possibilities of solving more sophisticated MFLP tasks. The conditions for the weights of branches, the capacity of centers and their combinations were newly included in the calculation of the location of centers. Furthermore, the conditions for the location of the center in the space and therefore whether or not the center should lie in a given polygon, which is determined by semiplanes, were included in the calculation. The last complex task solves the combination of all the above mentioned conditions. The application of this method was demonstrated in the military environment in supporting the commander's decision-making process in planning the deployment of logistical support for artillery units. All tasks indicate the expected results, thus the conditions were fulfilled, which were set out in the assignment. This method has its liabilities, which were explained in more detail in previous research. These liabilities result from the iterative process of generating random numbers and subsequently obtaining the resulting value of the location of the centers. Naturally, the evolutionary calculation, as Excel itself refers to it, is also challenging for HW equipment, which has an influence on the resulting values. For these reasons, 100 calculations were performed for each test to partially eliminate these effects. Assuming knowledge of these liabilities, it is then possible to obtain valid results, which can be in close proximity to the optimal solution. The article has the ambition to show the advantage of the solution of the optimization task MFLP with the help of the widely available Microsoft Excel software, where it is relatively easy to implement the limiting conditions of the solution and can be used in common practice.

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Primary Paper Section: K

Secondary Paper Section: KA, BB