CONDUCTING AN EXAMINATION OF THE TRAJECTORY AND WORKSPACE OF THE MANIPULATOR WITHIN THE MATLAB ENVIRONMENT

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Abstract: The paper delves into the possibilities of computer modeling for manipulator systems within the Matlab program. The analysis is directed towards a two-link manipulator model. Within the framework of kinematic analysis, the paper confronts the solution to the inverse kinematics problem and subsequently navigates through the solution to the direct kinematics problem. The trajectory is modeled by a fifth-degree polynomial during the motion from a known initial position to the specified final position within defined working regions of the manipulator. The work encompasses graphical representation of the results. The manipulator's workspace is also delineated, considering angular constraints on both arms and illustrating the trajectories during operational movement.

Keywords: manipulator, workspace, computer modeling, kinematic analysis

1 Introduction

The use of computer simulations is advancing alongside rapidly developing software and hardware designed for user applications in various engineering fields. Software simulations allow assessing the system's behavior during motion in work-related activities. Created animations provide a real-time overview of movement, allowing for retrospective adjustments to the designed model and retesting the model's functionality through a new simulation. Solution outputs in the form of graphs enable real-time monitoring of variables during the simulation. Computer modeling is renowned for its speed of execution and solution flexibility [1-4].

Computer modeling can be categorized based on the methodology of model creation and solution:

- Mathematical modeling, including software like Matlab, Maple, Mathematica
- Multibody modeling, utilizing software such as Matlab, Simulink, Simscape, MSC Adams

Mathematical modeling employs mathematical equations to define the properties of elements in the modeled system, expressing motion through derived equations and kinematic dependencies among model components. Due to the time-intensive nature of deriving motion equations and equations expressing kinematic dependencies, mathematical modeling is predominantly employed in solving simpler models with a lower number of degrees of freedom [5-7].

Multibody modeling differs from mathematical modeling in that it does not describe the motion equations of the modeled system. Various software options allow defining a multibody dynamic system - Multi Body System (MBS) - with a wide range of body shapes. By defining the geometry of bodies with various basic shapes, a solvable model can be created. This approach saves time, as there's no need to formulate motion equations. However, these programs demand robust hardware and the expertise, knowledge, and skills of the designer [7-9].

In the subsequent chapters of the contribution, the utilization of computer simulation in the Matlab program will be demonstrated.

2 The manipulator model

In the field of robotic systems kinematics, we encounter the solution of direct and inverse kinematics tasks. In this section, the solution to the direct and inverse kinematic tasks employs a model of a two-link robotic arm, as found, for instance, in Scara manipulators, an example of which is shown in Fig. 1 a) – b). The inverse kinematics task aims to determine the angular coordinates of actuators given the positional coordinates of the endpoint. In terms of kinematics, the analyzed Scara model is considered relatively simple and can be solved analytically. The solution approach will be based on the scheme of the two-link robotic arm in Fig. 2 a) - c), [1-3].

If we look at the model from the kinematics perspective, it represents an open kinematic chain in terms of structure. The individual links of the kinematic chain are connected by kinematic pairs. In this case, rotational kinematic pairs are employed. The model consists of two arms with lengths L_1=0.22 [m] a L_2=0.19 [m], anchored to a fixed base. This base could be either a stand, or the arms may be mounted on a mobile chassis, especially if intended for use as a service robot. A manipulator with a fixed base and two arms has 2 degrees of freedom in movement. We will investigate the motion of the two arms of the manipulator in a plane in Fig. 2 b) - c) [1-4].
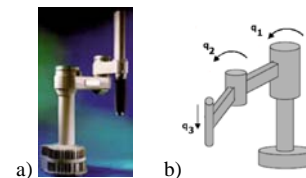

Figure 1. a) Scara manipulator, b)Scara manipulator model [1-3].
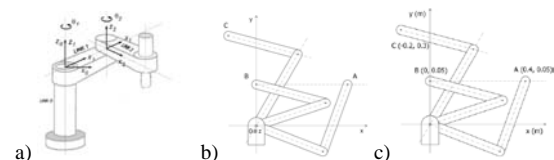

Figure 2. a) Scara manipulator [1-3], b) - c) two link manipulator model.

The objective of the study is to investigate the motion of the arms, specifically focusing on the movement of the endpoint of the second arm. These involve concurrent motions. Matrix methods, commonly encountered in the works of various authors [1-4], are advantageously employed in their resolution. The paper will further address the inverse and direct kinematics tasks. The rotation angle in each kinematic pair is denoted by angles Θ_1, Θ_2 as per Fig. 2a). Each element is assigned a coordinate system O_i, x_i, y_i, z_i and each joint is assigned a generalized coordinate q_i, defined along the axis of rotation (Fig. 2a). Generalized coordinates determining the instantaneous position of the body are denoted q_1, q_2 (Fig. 3). It holds true for generalized coordinates that $q_1 = \Theta_1$, $q_2 = \Theta_2$. We assume that the actuators are situated in rotational kinematic pairs.
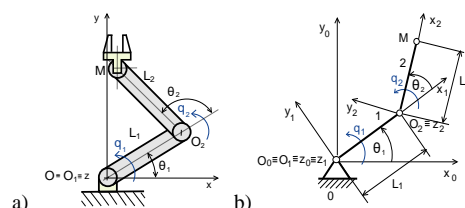

Figure 3. a)-b) Model of the manipulator with generalized coordinates $q_1 = \theta_1$, $q_2 = \theta_2$ in joint.

Before addressing the direct kinematics problem to determine the position of the end effector of the arm, we need to know the rotation angles for the given arm configuration. Therefore, the inverse kinematics problem is initially solved. Considering the desired location of the end point M of the robotic arm, denoted by x_M, y_M, we aim to determine the joint rotation angles that position the endpoint M at the specified location, as shown in Fig. 4. The positions of points are listed in Table 1.

The position of the end effector M is described by the equations:

$$x_M = L_1 cos\theta_1 + L_2 cos\,(\theta_1 + \theta_2) \qquad (1)$$
$$y_M = L_1 sin\theta_1 + L_2 sin\,(\theta_1 + \theta_2) \qquad (2)$$

To determine the angles, we will employ the equations (1) and (2) provided below. Typically, there is more than one solution, as evident in Table 2. The task will be addressed concerning the movement of the end point of the second arm, traversing different positions denoted as A, B, C, with their respective coordinates listed in Table 1.

Table 1. Coordinates x, y of the points A, B, C

	A	B	C
x (m)	0.4	0	-0.2
y (m)	0.05	0.05	0.3

For the given position x_M and y_M (Tab.1), the solution involves determining the values of the two unknown angles, θ_1 a θ_2, through the resolution of equations (1) and (2). This constitutes the resolution of the inverse kinematics problem. In MATLAB, this is achieved using the following script:

```
x0=0.400    %M0
y0=0.050
xtf=0       %Mtf
ytf=0.050
L1=0.220
L2=0.190
syms theta10 theta20 theta1tf theta2tf
E10 = L1*cos(theta10)+L2*cos(theta10+theta20)-x0;
E20 = L1*sin(theta10)+L2*sin(theta10+theta20)-y0;
[theta10, theta20] = solve(E10,E20);
theta10 = double(theta10*(180/pi))
theta20 = double(theta20*(180/pi))
E1tf = L1*cos(theta1tf)+L2*cos(theta1tf+theta2tf)-xtf;
E2tf = L1*sin(theta1tf)+L2*sin(theta1tf+theta2tf)-ytf;
[theta1tf, theta2tf] = solve(E1tf,E2tf);
theta1tf = double(theta1tf*(180/pi))
theta2tf = double(theta2tf*(180/pi))
```

We calculate the angles of the arms in the initial position of point M_0 x_{M0} and y_{M0} and determine the angles θ_{10} a θ_{20}. Subsequently, we ascertain the angles of the arms in the final position of point M_{1tf} x_{M1tf} and y_{M1tf} and identify the angles θ_{1tf} a θ_{2tf}, as presented in Table 2.

The angles' magnitudes during the movement of the endpoint along the trajectory between points A-B and B-C are detailed in Table 2.

Table 2. Sizes of angles between points A-B, B-C

	A-B	B-C
Θ_{10}	16.8898	137.7221
	-2.6398	42.2779
Θ_{20}	-21.0903	-168.7723
	21.0903	168.7723
Θ_{1tf}	137.7221	149.9268
	42.2779	97.4534
Θ_{2tf}	-168.7723	-57.0260
	168.7723	57.0260

We will use the angle values θ_{10}, θ_{20}, θ_{1tf}, θ_{2tf} from Table 2 at the defined points A-B, B-C to plot the respective trajectories k_1, k_2 and k_3 in Table 3.

Table 3. Sizes of angles between points A-B trajectory k_1, B-C trajectory k_2 and k_3

	A-B (k_1)	B-C (k_2)	B-C (k_3)
Θ_{10}	-2.6398	137.7221	137.7221

Θ_{20}	21.0903	-168.7723	-168.7723
Θ_{1tf}	42.2779	149.9268	97.4534
Θ_{2tf}	168.7723	-57.0260	57.0260

An illustration of the arm positions at point B at time t_0 and point C at time t_{fin} can be seen in Fig. 4. The trajectory of the end point during its movement from point B to point C is also depicted in Fig. 4.
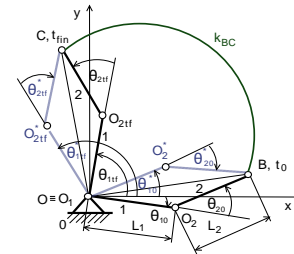

Figure 4. Manipulator model with two arms, displaying angles θ_{1t0} , θ_{2t0} , θ_{1tfin} , θ_{2tfin}, and the trajectory k_{BC}.

Determine the trajectories along which the endpoint will move between the defined points, which will be addressed by solving direct kinematics in the following section.

3 Computer simulation and trajectory

In the Matlab program, we determine the trajectory during the motion of the arms from a position defined by the endpoint of the second arm, starting at point A and ending at point B, using the calculated angles in Table 3. A 5th-degree polynomial will be employed in this process. The solution will yield sequential trajectories k_1, k_2 a k_3, illustrated in subsequent figures [5-9].

Trajectory k_1 corresponds to the movement of the endpoint from point A to point B (Tab. 1) with initial and final angles given in Tab. 3, as shown in Fig. 5.
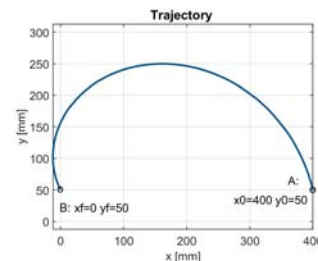

Figure 5. Trajectory k_1 between points A-B.

Along trajectory k_1, between A and B, the endpoint of the second arm of the manipulator will move, following specified angles of rotation, angular velocity, and angular acceleration, as depicted in Fig. 6.
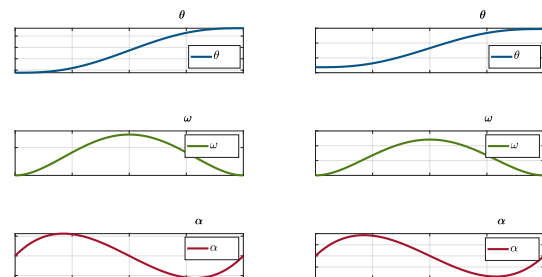

Figure 6. Movement characteristics of the manipulator with trajectory k_1 from point A to point B.

Figure 7 a) illustrates the workspace of the mentioned manipulator Scara. Figure 7 b) illustrates the workspace with angular constraints $-45° \leq \theta_1 \leq 225°$ and $0° \leq \theta_2 \leq 180°$ and the trajectory between points A and B. Fig. 1.
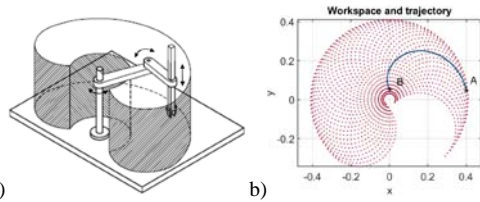
Figure 7. a) The workspace of the manipulator Scara, b) trajectory k_1 between A-B and the manipulator workspace.

The trajectory k_2 from point B to point C, based on the angles listed in Table 3, is shown in Fig. 8 a). The determined trajectory k_3 based on the angles in Table 3 is depicted in Fig. 8 b).
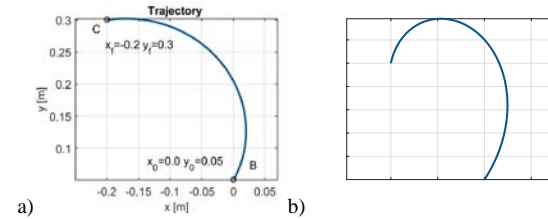

Figure 8. Trajectory a) k_2 between points B-C, b) k_3 between points B-C.

Figure 9 displays graphs depicting the kinematic parameters of the manipulator as the endpoint moves along trajectory k_2 from point B to point C.
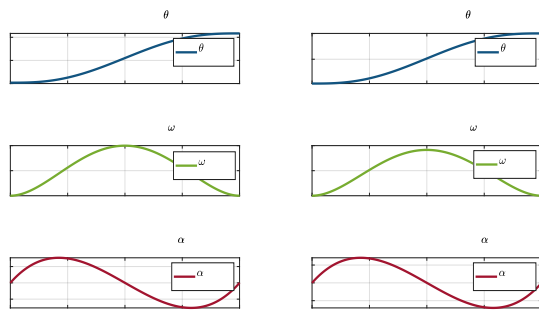

Figure 9. Kinematic parameters of the manipulator for k_2 between B-C.

Figure 10 displays graphs illustrating the kinematic parameters of the manipulator as the endpoint moves along trajectory k_3 from point B to point C.
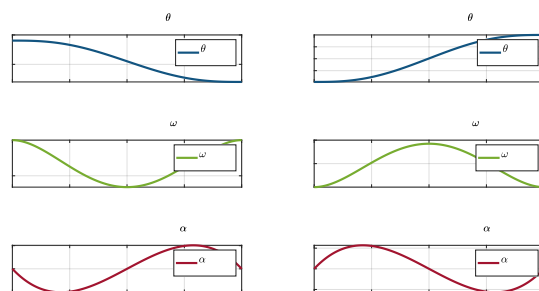

Figure 10. Kinematic parameters of the manipulator for k_3 between B-C.

After solving the forward kinematics in Matlab, Fig. 11 displays the workspace with trajectories k_1, k_2, k_3 , where:

a) $-45° \leq \theta_1 \leq 225°$ and $0° \leq \theta_2 \leq 180°$,
b) $-45° \leq \theta_1 \leq 180°$ and $0° \leq \theta_2 \leq 180°$,
c) $-45° \leq \theta_1 \leq 105°$ and $0° \leq \theta_2 \leq 180°$,
d) $-20° \leq \theta_1 \leq 105°$ and $0° \leq \theta_2 \leq 180°$,
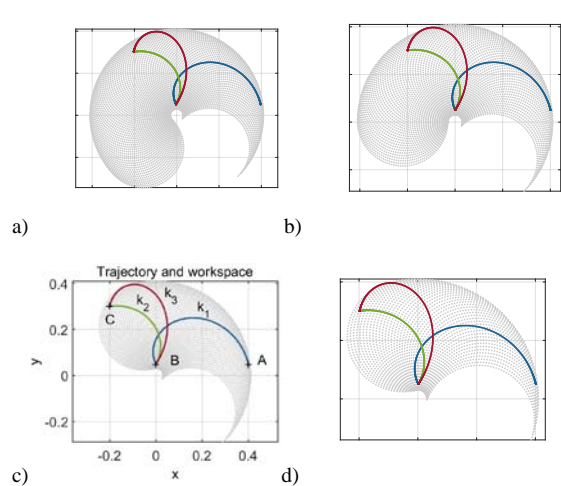

Figure 11. a) - d) Trajectories k_1, k_2, k_3 and the workspace of the manipulator.

Computer modeling of a prototype is a convenient tool for creating, processing, modifying, and presenting simulation results in the form of graphs. Graphs of output variables allow real-time monitoring of measured values during the simulation [5-9].

In multibody modeling software, there is no need to solve the mathematically described motion with a motion equation. We only use information about the geometric parameters of the designed model. An example of a manipulator model is shown in Fig. 11 a) to d), featuring various perspectives of the simulation model in the MSC Adams View software during "multibody modeling" with illustrated end-point trajectories.
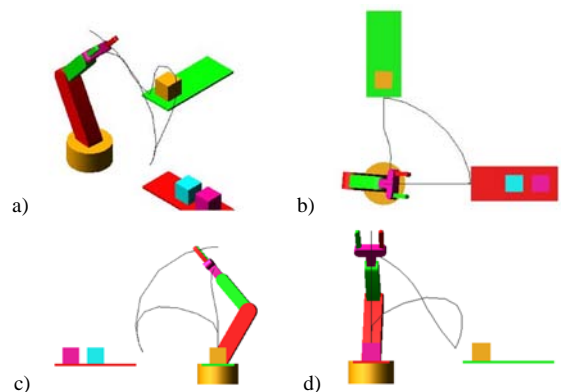

Figure 12. a) - d) The model in motion depicted using MSC Adams View software.

Further works by the authors of the paper are dedicated to modeling in MSC Adams [5-9].

4 Conclusion

Simulation software serves as a useful design tool, saving time and resources. It is also suitable for in-depth research of mechanical systems.

The advantage of computer simulation lies in the ability to simulate the motion of a prototype model and obtain information about the magnitudes of the required kinematic and force variables of the model. Based on simulation results, it is possible to construct an actual model and design drives. When designing drives for a mechanical system, it is essential to consider the maximum forces when manipulating various loads. Hence, designing a drive with specific parameters for the intended use of the manipulator is necessary [10-14].

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