SYSTEMS MODELLING: QUEUING SYSTEM TO OPTIMIZE WORKLOAD

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Abstract: For pharmacists to fulfil their roles in the provision of pharmaceutical healthcare, it is necessary to ensure the effective organization of the work of individual healthcare professionals. We performed measurements at a selected pharmacy, which, among other things, allowed us to determine the time between people arriving and the average number of pharmaceuticals purchased. Using the available measurement data, we created a queueing system that can simulate the operation of a pharmacy. The system can save time for other pharmacists to fully devote themselves to other activities related to the provision of pharmaceutical healthcare, such as consultation, preparation of medicines in the laboratory, back-checking of prescriptions, or stock control.

Keywords: queueing system; pharmacy; modeling; simulation

1 Introduction

The term pharmacy is derived from the Greek word pharmakon. This word denoted a medicinal/poisonous substance or a magical remedy. We can say that pharmacy is a system of pharmaceutical industries that include Pharmaceutical education, Pharmacy, Pharmaceutical research, Pharmaceutical production, Pharmaceutical control, Pharmaceutical wholesale distribution, Historiography, and Organization and management of pharmacy [1]. Community pharmacies are easily accessible healthcare facilities that provide pharmaceutical healthcare for patients. A pharmacist is both a medicine expert and the end link in healthcare delivery who can control the patient's treatment process. All registered medicines are effective, high quality, and safe and their use is essential for improving patients' health outcomes. Medicines are a tool for healthcare delivery. They have not only therapeutic but also social, ethical, and economic value [2,3,4,5,6].

Like medicine and pharmacy, the profession of a pharmacist has undergone many changes in the historical development of society. In prehistoric times, the function of a physician and pharmacist was performed by a healer or shaman. He diagnosed diseases, suggested treatment procedures (empirical knowledge about treatment), searched for suitable drugs (plants, animal products, minerals), and prepared medicines from them. In the period of antiquity, a separate profession of root cutters was separated from the profession of a healer. This began to form the beginnings of an independent pharmaceutical function. Later, other separate professions were carved out of the healing profession for example manufacturers of ointments and sellers of medicines, aromatic waters, spices, and plants. In the ancient world was significant for example medicine and pharmacy in Ancient Greece, Rome, Egypt, India and, China. The first community pharmacies were established in the territory of the Arab Empire during the Middle Ages. At the same time as community pharmacies, pharmacy literature was created. Requirements for pharmacy equipment and ethical principles for the practice of the profession of pharmacist were also developed. From the 11th century, community pharmacies were also created in Europe. For several centuries, they were the only representative of pharmacy in society. Pharmaceutical research, education of apprentices, pharmaceutical control, preparation of medicines, and dispensing took place in community pharmacies. This period came to an end in the 19th century, when separate pharmaceutical industries were carved out of the pharmacy environment, such as Pharmaceutical education, Pharmacy, Pharmaceutical research, Pharmaceutical production, Pharmaceutical control, and Pharmaceutical wholesale distribution [1,4,7].

The role of pharmacists in society has also undergone significant historical development. From artisan and manufacturer to a teacher in apprenticeship training, trader and scientist to the current perception of the pharmacist as a consultant to the doctor and advisor to the patient. The change in the role of pharmacists in society has also contributed to the change in the orientation of pharmacy in the 20th century from a primary interest in drugs and medicines to an interest in the provision of individualized pharmacotherapy to the individual patient (pharmaceutical care, personalized medication therapies) This is why pharmacists have a greater responsibility to improve their knowledge. Pharmaceutical care leads to improvement in health outcomes and cost-effective therapy [4,8,9]. Some of the activities carried out by pharmacists in the context of dispensing medicines or in the context of pharmacy consultation activities include checking for interactions, contraindications, and duplications within the medicines being taken, checking for errors on prescriptions, and providing information on the correct use of medicines and on the risks of pharmacotherapy, for example, in the form of side effects, on the proper storage of medicines, on the management of unused medicinal products. They also include health promotion, disease prevention, and chronic disease management. In addition to dispensing medicines, pharmacists in community pharmacies may also provide consultation services to patients or other health professionals. Consultation activities may focus, for example, on providing information on the correct and safe use of medicines, on resolving medication problems, on recording adverse reactions, or on informing about healthy lifestyle options [3,8,10,11]. Effective verbal and nonverbal communication with the patient or other healthcare professionals is essential in the practice of pharmacists. This includes, for example, using simple and easy-to-understand terms that the patient can easily understand and checking that the patient has understood the information correctly by asking follow-up questions. It is also necessary to pay attention to the patient's non-verbal expressions [12,13]. Pharmacists had also important roles during the COVID-19 pandemic, working alongside other health professionals on the frontline of patient care. Pharmacies have adapted pharmaceutical care to new needs and challenges, such as the preparation and delivery of disinfectants, information, and counselling for COVID-19 patients, or the introduction of home drug delivery systems [14].

The development of information and communication technologies and their implementation in practice has enabled the introduction of eHealth and ePrescription systems in pharmacies. These systems have been used in practice for a long time and are applied not only in the Slovak Republic but also in many other countries of the world. In the Slovak Republic, the eHealth system has been introduced since 1 January 2018 to improve the quality of healthcare provided. Electronic prescribing and dispensing of medicines, medical devices, or dietetic food, Patient´s Electronic Health Record, eExamination, and other system functions are currently used in practice. The benefits of introducing ePrescribing into practice include, for example, improving the quality of healthcare provided, increasing the efficiency and effectiveness of prescribing and dispensing medicines, reducing prescribing errors, reducing healthcare costs, increasing patient safety, preventing adverse drug reactions, preventing the occurrence of fraudulent prescriptions, reducing over-prescribing and, most importantly, saving time for doctors, pharmacists, and patients. An electronic prescription is the equivalent of a paper prescription. The stamp and signature of the prescribing doctor are replaced in the electronic prescription by the electronic passport of the healthcare professional. The system performs a background check for possible drug interactions or duplications in prescribed medications. Once the patient is identified by presenting his/her insurance card or eID card with a chip at the pharmacy, the system will show the pharmacist all the prescribed medicines, medical devices, and dietetic food that have not been dispensed [15,16].

Community pharmacists are among the key health professionals who contribute to the health of the individuals and communities they serve, while strengthening Europe's health systems. For pharmacists to have enough time for patients and to be able to give them their full attention, it is necessary to streamline and improve the organisation of work in the pharmacy. The launch of eHealth can contribute to this goal, as well as programmes and applications enabling the streamlining and improvement of the organisation of work in the pharmacy by taking into account the average number of patients in the pharmacy during the opening hours of the pharmacy. The saved time that pharmacists gain through the implementation of programs and applications aimed at better organization and streamlining of their work in dispensing medicines can then be used for consultations with specific patients regarding the solution of drug problems, control of interactions, contraindications, duplications in the medicines taken, to provide information on the correct and safe use of medicines, on the risks of pharmacotherapy, adverse effects and how to record them, on the correct storage of medicines, on the disposal of unused and overexpired medicines, on the possibilities of health promotion, healthy lifestyles, disease prevention. They may also cover the management of chronic diseases. Changes in the organization of pharmacists' work could contribute to the fulfilment of the Pharmacy 2030: A vision for Community Pharmacy in Europe. Pharmacy 2030 is the vision of a European association called the Pharmaceutical Group of the European Union (PGEU) whose role is to represent community pharmacies in relation to legislative and policy initiatives at EU level that affect pharmacists and public health. It is aimed at developing the pharmacy profession and responding to current challenges in healthcare. It also seeks to promote the maximum contribution of pharmacists in the field of patient pharmacotherapy and its safety, but also in the context of medication adherence interventions [8]. These changes in the organization of pharmacists' work may also have a positive impact on the role of pharmacists in society, not only in relation to the pharmacotherapy of patients but also in the field of public health protection.

2 Theoretical background

The Danish mathematician A. K. Erlang founded the queuing theory in the early 1900s with his publication "The Theory of Probabilities and Telephone Conversations". The problem that researches at the time was trying to solve was how to manage and optimize the telecommunications network.
Queuing systems have a server unit(s) (e.g. cashier in the store) which provides service to the entities (e.g. customers) waiting to be served (See “Figure 1”). The number of entities can be served per unit of time is the capacity of the serving channel (characterized by μ), and the number of individuals to be served in a unit of time is the intensity of arrival of individuals (characterized by λ).

Figure 1. Single server queuing system

The serving intensity is derived from the time intervals between the two outgoing demands with a certain distribution. In this case, $\mu = 1/t_0$, where t_0 is the mean of the service time. Also valid: $t_0 = 1/\mu$. We specify the μ parameter in units of time, e.g. hours, minutes, days. The intensity of the arrival, i.e. λ, is obtained by observation or experimentation. In general, the time intervals between consecutive requests are described using a probability distribution. If queuing system serves μ claim in one unit of time, then serving one claim *is 1/μ.* If the demands come into the system with λ intensity, the time between each incoming request *is* $1/$. Based on this, we can define the following parameter: $= \lambda/\mu$, which specifies the utilization factor (traffic intensity), that is, the average occupancy of the serving channel.

Service times and the times between arrivals can be constant, variable, or random. Typical applications of Queuing systems include e.g. telecommunications systems, computer networks, Internet communications, transport systems (road, rail), airports, ports, repair shops, banks and post offices, hospitals, specialist clinics, manufacturing processes, intersections, deliveries and many, many other similar systems. From this we can see that queuing systems are also encountered on a daily basis.

Queuing systems examinations can be divided into two large groups: Analytical solutions, where, based on the known parameters of the model, we calculate or estimate the additional parameters of the model using probabilistic or mathematical tools. e.g. queue length, number of requests in the system and etc. This can be used in simple cases.

Modeling and simulations, when we simulate a case based on the known parameters of the model, or cases and from these we obtain an estimate of the model's behavior. This is mainly used for complex systems.

The stability of the system will be examined in terms of the parameter ρ. As stated above: $\rho = \lambda/\mu$. There are three basic cases:

$$\rho = 1$$

For deterministic systems, this is the ideal case. If we assume that we have a serving unit, then this serving unit is continuously operating, and no queue is created. The moment one demand is served, another is just arriving.

$$\rho < 1$$

In this case, requests arrive more slowly and are served more quickly. Such a system is stable, the serving unit will occasionally stop working because there are no pending requests in the system at times. It follows that no queue is generated in such systems.

$$\rho > 1$$

In such systems, requests arrive more quickly and are served more slowly. The queue will grow until it reaches infinity. Such systems are not stable systems. The solution to stability is to limit the queue length. The simplest case is when incoming requests that do not fit in the queue are automatically rejected.
We should also mention that fully deterministic systems are rarely encountered in practice. More common are cases where the arrival is stochastic and the service deterministic, or the arrival is deterministic and the service stochastic.

Systems with multiple server units are similar to the systems discussed above, but instead of the parameter ρ we use the parameter α: $\alpha = \lambda/n\mu$, where n is the number of server units (See “Figure 2”).

Figure 2. Queuing system with n server

The service principles may be:

FIFO - (First In First Out): if a request arrives in the system and at least one of the serving units is empty, it will be served immediately. If, on the other hand, the serving units are not empty - occupied, then the requests are queued and served in the order in which they came in.

LIFO - (Last In First Out): this principle is similar to the previous one, but the last incoming request is served first and so on.

SIRO - (Selection in Random Order): Requests waiting in the queue are randomly selected for service.

FCFS and LCFS - (First Come First Served and Last Come First Served): First Come First Served and Last Come First Served are served first. It is very similar to FIFO and LIFO, and the result is often virtually the same, and may even be used synonymously, but there is a difference. Consider a queuing system with parallel server units in the figure above. If we assume that the serving time can vary, then the FIFO principle should not occur, but the FCFS principle should be satisfied.

PRI - (Priority). In these systems, demands are different, with a finite number M of priority indices. The priority can be:

Relative priority: when a serving unit is released, the higher priority demand is served. Equal priorities are queued and selected according to one of the principles mentioned above.
Absolute priority: If another higher priority request enters the system while a request is being serviced, the request being serviced is terminated - it is interrupted, returned to the queue and the higher priority request is serviced immediately.

Time-shared serving: nowadays used mainly in computer processing, but also in telecommunications and where serving is done on high performance and high traffic server units. An incoming request is only partially serviced at a time and then returned to the queue so that another request can be partially serviced. This is repeated cyclically until the requests are fully serviced. There is some feedback between the service and the queue.

GI - (General Discipline): means Free Discipline, i.e. there is no service principle for the queued requests. This principle is also found in many literatures, but it is very rare in practice and makes a numerical or analytical approach difficult. [17]

3 Result

In this work, simulations were created in MATLAB software using the SimEvents library. The data were sorted and arranged in Microsoft Excel. The visualizations were also created using the aforementioned software, which was greatly assisted by the article [18].

3.1. Simulate arrival times

Our goal is to quantify the measurements with the help of a curve that similar to the measured data as best as possible. To determine the coefficients of the approximation polynomial, we used the least squares method (hereinafter referred to as LSM), for which the algorithm is shown in Figure 3. [19].

```
clear all, clc
x = [0  10 ...];
y = [0.1  0.2 ...];
m=5;
w=ones(size(x));
A=zeros(m+1,m+1);
I=ones(1,length(x));
for i=1:m+1
   Y(i)=(y.*x.^(i-1))*w';
   for j=1:m+1
      A(i,j)=x.^(i-1+j-1)*w';
   end
end
a=(A\Y')
```

Figure 3. LMS MATLAB algorithm

With the program, we were able to determine the coefficients of a fifth order polynomial that is sufficiently representative of the measured data (See “Figure 4”).

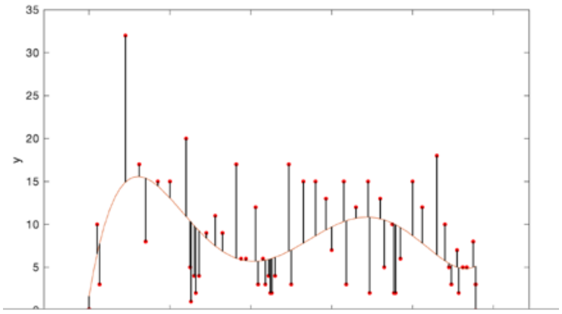

Figure 4. Obtained fifth order polynomial

From the coefficients obtained as the output of the program, we can write the polynomial, which is:

$$f(x) = 5.6923 \cdot 10^{-11}x^5 - 7.6377 \cdot 10^{-8}x^4 + 3.6635 \cdot 10^{-5}x^3 - 0.0074x^2 + 0.5626x + 1.6366$$

In Simevents, with the help of the Entity Generator block, it is possible to create entities, in our case to simulate the arrival of customers at the pharmacy (See “Figure 5”). To determine the time elapsed between the arrivals of each customer, we use the previously mentioned polynomial, which is embedded in a MATLAB function block. The independent variable of the function is determined by simulation time. This block is located in the right branch of a fork, which is marked in red in the figure. In order to simulate the possible overtime, it was necessary to simulate this condition. The true branch provides the entity generating block with the arrival times using the polynomial defined above, while the false branch stops the arrival in the system. In our case, we set an overtime of half an hour, while the total working time was set at 8 hours, i.e. 480 minutes.

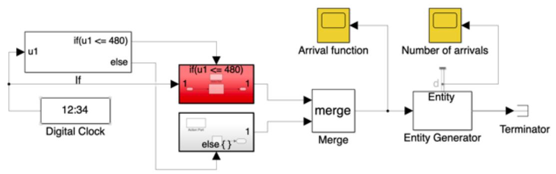

Figure 5. Entity generation with Simevents

With this method, it was possible to achieve results similar to real data (See “Figure 6”). It is clear from the simulation output that there are roughly 60 customers a day, which coincides with the measured results.

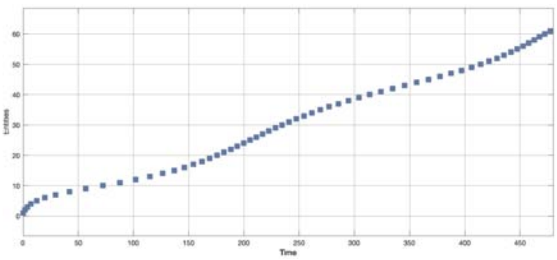

Figure 6. Arrival of customers to the pharmacy during the entire opening hours

3.2. Determining service times

The measured data were used to determine the average amount of medicines a customer buys (See “Figure 7”). For the resulting proportions, we determined the most ideal distribution curve, which in our case was the exponential distribution with a mean of 4,06.

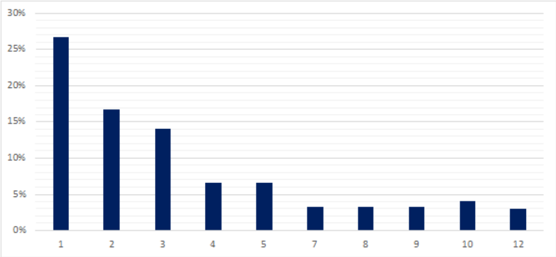

Figure 7. Distribution of the number of purchased pharmaceuticals

To determine the service time of each customer, i.e. how much time they will spend at the checkout, we had to measure how much of each pharmaceutical they buy. From this data, we have determined the frequency with which each item is purchased (See “Table 1”). This is a table. Tables should be placed in the main text near to the first time.

Table 1. Frequency of buying pharmacy

Pharmaceutical	Frequency	Average service time
Paralen 500	8,85%	42s
Carbo medicinalis	5,31%	25s
Helicid 40	5,31%	72s
Anopyrin 100 mg	4,42%	40s
⋮	⋮	⋮
Condrosulf 800 tbl	0,88%	60s

To determine the final service time, we derive discrete values using the exponential distribution mentioned above. The resulting value (Let n be) determines how many medicines the customer will buy. To calculate the service time, choose n medicines from Table 1 and add up their average service time.

3.3. Single server solution

Almost all components are available to create the final simulation environment. First, let’s look at the case of a checkout in a pharmacy (See “Figure 8”). Entities entering the system are generated as described above. Customers are line up in a FIFO queue of infinite length after arrival. In this case, we did not consider the length of the queue, as in reality there is no limit to the number of people waiting in front of a pharmacy or inside a pharmacy. From the queue, customers move on to a single server, which represent a checkout. The service time is determined as described above. Once served, the entities will be terminated.

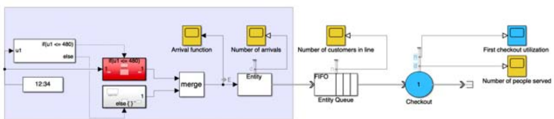

Figure 8. Single server solution

Sub result

No surprising values were obtained from the simulation results. The system worked as expected. In the pharmacy under study, two cash registers operate most of the time, so in our case the expected outcome was that the queue would become saturated, and the single server would not be able to serve all participants by the end of the day (See “Figure 9”). During the first half of the day, the cashier was able to service the incoming requests, but as we reached the midday rush hour, the queue started to fill up, and it didn't empty until the end of the day.

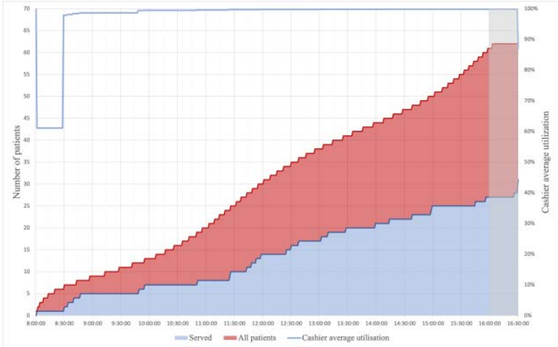

Figure 9. Single server solution result

3.4. Three server solution

As we have seen in the previous section, one checkout cannot deal with the demands that come into the system, so it is necessary to add new server units. For the pharmacy operator, it is not optimal to have unused checkout open, so we open and close them based on a decision system (See “Figure 10”). The checkout is opened when the number of queues rises above a certain value and closed when it falls below another value. These thresholds are monitored by a "Hit Crossing" block, which is used to switch an SR gate. These gate outputs are used to control an "Entity Gate" block, which corresponds to the opening and closing of the checkout.

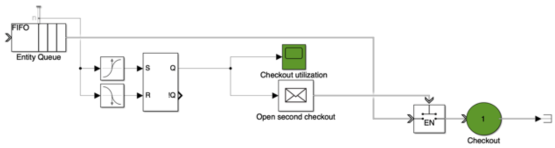

Figure 10. Control logic for open and close the checkout

The simulation was built with three checkouts (See “Figure 11”). The main cash desk is open throughout the opening hours. The second cashier is opened when there are at least 4 people in the queue and closed when there is at maximum 1, while for the third cashier these numbers are set to 8 and 4.

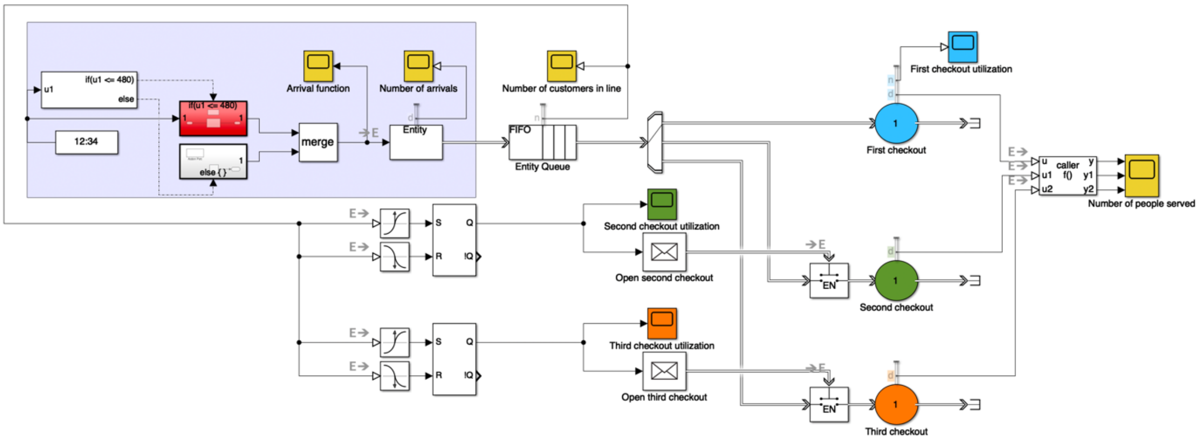

Figure 2. Three server unit solution with automatic unit controller

Final result

On the polynomial defining the demand arrivals into the system, it can be clearly observed that there are three periods when the time between each arrival is small. The first of these is the morning after opening, followed by a quiet period. During this period, even one server can cope with the demand (See "Figure. 12"). The second busy period lasts much longer and is followed by a less calm period. This is when the second cashier opens and will remain open until the end of the day. The two checkouts serve customers with satisfactory performance until the last rush before closing time. This is when the third cashier opens, and the last customer is served 30 minutes after closing time.

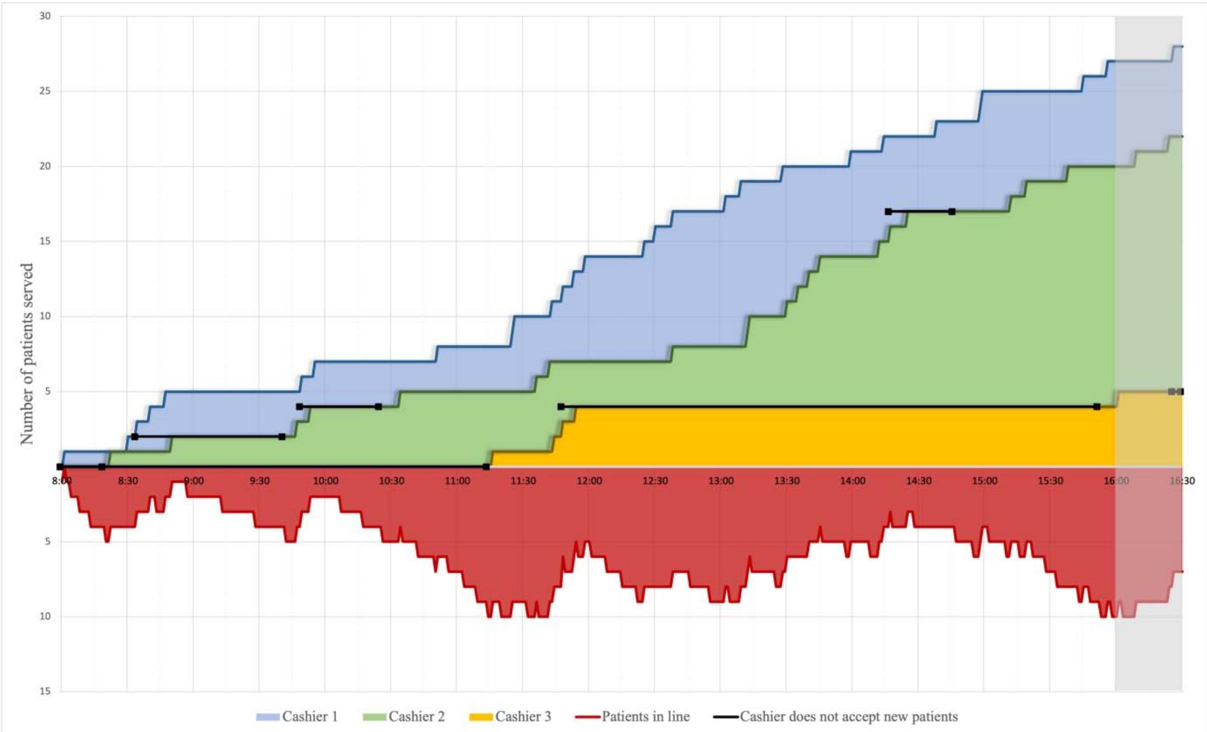

Figure. 3. Three server solution result

4 Discussion

The model we have built can represent the operation of a pharmacy with sufficient accuracy. The inputs to the tool can be modified to represent other institutions. The measured time span can be extended or contracted, but in all these cases the polynomial that generates the inputs must be modified. The model is a suitable tool for optimizing the working time of individual pharmacists. It is also good for the institution from an economic point of view, since it does not have to pay unnecessary staff, and for the employees, since they know when they have to work in the cashier's office or do other tasks. In the model presented, each cashier is opened and closed according to different thresholds, which can be modified at our discretion. This ensures that the model can be parameterized according to different needs. Optimization of these values is an option for further development.

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