BAYESIAN ESTIMATION OF REAL BUSINESS CYCLE MODEL: THE CASE OF POLAND

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Abstract: This paper presents the result of Bayesian estimation of real business cycle model augmented with mechanism of indivisible labor. We evaluate the structural parameters using quarterly data for Poland from 1997 to 2011. Methodology of the estimation is based on Bayes theorem. We use Kalman filter to estimate likelihood function and Metropolis algorithm to obtain posterior distribution for structural parameters. Our results are as follows. Firstly, we find that all estimated parameters are significantly greater than zero. We also find significantly greater value of the elasticity of output with respect to capital stock than it is assumed in standard calibration. The fit of model is relatively good taking under consideration model's simplicity. Moreover, the analysis of model's dynamics shows that capital stock and consumption are highly persistent in response to positive shock in technology.

Keywords: Real Business Cycle, Bayesian estimation.

1 Introduction

The Real Business Cycle model (hereafter RBC) is one of the cornerstones of the modern macroeconomics. It analyzes the aggregate dynamics of economy as the result of the optimal microeconomic decisions taken by individual agents, who tend to maximize their stream of current and discounted future utilities subject to aggregate dynamics and resource constraint of the economy. Moreover, the framework of the RBC model includes uncertainty, as a result of the stochastic random process (a structural shock), so agents can only expect future values of economic variables. It is assumed that they expect them in rational way, using the information set of all current and past realization of endogenous variables. In the RBC it is also assumed that all prices, including goods and factors, are fully flexible and all markets are in equilibrium [see e.g.: Kydland, Prescott, 1982; Long, Plosser, 1983; King, Plosser, Rebelo, 1988; Plosser, 1989; Stadler, 1994].

This paper presents the results of the estimation of the RBC model for a Polish economy. We choose Hansen's real business cycle model [Hansen, 1985] which contains the mechanism of indivisible labor. Such model has good empirical performance in comparison with other standard RBC models presented in the literature [see Hansen, Wright, 1992]. To estimate structural parameters of the model we use Bayesian approach which is recently more often used to analyze dynamic stochastic general equilibrium models (DSGE) [1] .

The rest of the paper is organized as follows. Section 2 presents theoretical model. We only focus on log-linear approximation of the equilibrium conditions. All variables are expressed as the percent deviation from the steady-state. Section 3 discusses Bayesian techniques and statistical data that have been used to estimate structural parameters. Section 4 contains results of estimation including: the posterior distributions of estimated parameters, the fit of the model and the posterior distributions of the impulse response functions for endogenous variables which present expected dynamics of the model after including uncertainty about estimated parameters. Section 5 concludes.

2 Structure of the model

From the beginning of their existence, the real business cycle models have had serious problems with fit to labor market mechanism. Observed in the data relative high variation in hours compared to output couldn't have been explained by the RBC without making an assumption of high elasticity of labor supply. However, high level of elasticity was at odds with microeconomic evidence [Hansen, 1985, p. 305; Mankiw, 1989, p. 85 – 86; Summers, 1986, p. 24].

In considered model the above-mentioned problem is omitted by assumption that labor is indivisible which implies that individuals can only work full-time or be unemployed. Therefore agents do not choose amount of labor which they supply to the market, but a contract with a probability of working full-time. The random variable, called an employment lottery, decides which agent will work. Moreover, complete market of state-contingent securities against unemployment is included in the model. All agents are then identical so we can analyze social planner problem [Hansen 1985, p. 316-317; McCandless, 2008, p. 112-113]. Implementation of indivisible labor also implies that preferences of agent are described by the quasi-linear utility function with decreasing marginal utility of consumption and constant marginal disutility of labor.

According to Hansen's model, output (y_t) is generated by using two factors: physical capital (k_t) and labor effort (h_t). We assume that all firms have access to the identical Cobb-Douglas production technology with constant returns to scale. This function can be expressed, in log-linear terms, as[2]:

$$y_t = z_t + \alpha k_{t-1} + (1-\alpha)h_t \qquad (1)$$

where: $\alpha \in (0;1)$ is the elasticity of output with respect to capital, and z_t represents level of technology which follows a stationary autoregressive process:

$$z_t = \rho z_{t-1} + \varepsilon_t \qquad (2)$$

where: $\rho \in (0;1)$ is an autoregressive parameter and $\varepsilon_t \sim i.i.d.N(0,\sigma^2)$ denotes the technological innovation.

Moreover, the single good has to be either invested (i_t) or consumed (c_t), so in the log-linear terms following resource constrain must be satisfied:

$$y_t = \frac{c}{y}c_t + \frac{i}{y}i_t \qquad (3)$$

where: $\frac{c}{y}$ and $\frac{i}{y}$ are shares of consumption and investment in output in steady-state respectively.

In the model capital stock evolves according to standard law of motion which in log-linear terms is given by:

$$k_t = (1-\delta)k_{t-1} + \delta i_t \qquad (4)$$

where $\delta \in (0;1)$ is the rate of capital depreciation.

The preferences of the representative household are described by utility function which is log-linear in consumption and linear in labor effort. Moreover, representative household tends to maximize the expected value of the stream of current and discounted future utilities subject to: (i) production function, (ii) law of motion for capital stock and (iii) resource constraint with respect to: consumption, labor and future capital stock under no-Ponzi-game condition. The solution of this problem yields first order conditions which consist of the Euler equation and the labor supply equation.

The Euler equation equals the marginal cost in terms of utility of investing single additional unit of good into capital stock with the expected marginal utility gain in the next period, while the labor supply equation implies that the marginal rate of substitution between labor and consumption must be equal the marginal product of labor. These equations can be expressed in log-linear terms as:

[1] The recent examples of papers which use Bayesian techniques to estimate DSGE model are e.g. Schorfheide [2000], Smets, Wouters [2003; 2007] or Rabanal and Rubio-Ramirez [2005].

[2] For technical details on derivation of real business cycle model (with or without indivisible labor) see, among others, McCandless [2008], Hatrley, Hoover, Salyer [2006] or Kuchta, Piłat [2010].

$$c_t = E_t(c_{t+1}) - [1-\beta(1-\delta)]E_t(y_{t+1}-k_t) \quad (5)$$

$$c_t = y_t - h_t \quad (6)$$

where E_t denotes the expectation operator conditional on information available at time t and $\beta \in (0;1)$ is the discount factor.

3 Method of estimation and data

In this part we present the method of estimation which is used to estimate structural parameters of the RBC model. The procedure of estimation consists of several steps[3]. In the first step the model (equation (1)-(6)) is solved using perturbation method based on first order approximation of the policy and transition functions [Schmitt-Grohe, Uribe, 2004]. The solution of model can be interpreted as the transition equation in the state-space representation of the DSGE model. In the second step we use empirical time-series to construct measurement equation to obtain state-space model. In the next step the Kalman filter is used to evaluate likelihood function [see Hamilton, 1994, p. 372-409; Canova, 2007, p. 214-220; Commandeur, Koopman, 2007]. After that we can use Bayes theorem to construct posterior distribution of parameters of interest according to the formula below [Fernandez-Villaverde, 2010, p. 9]:

$$p(\theta \mid x^T) = \frac{p(\theta)p(x^T \mid \theta)}{\int p(\theta)p(x^T \mid \theta)d\theta} \quad (7)$$

where: $p(\theta \mid x^T)$ represents posterior distribution, $p(\theta)$ is the prior distribution, $p(x^T \mid \theta)$ is the likelihood function and $\int p(\theta)p(x^T \mid \theta)d\theta$ denotes marginal density of data.

To obtain the posterior distribution of parameters we apply Metropolis [Metropolis, et. al., 1953] algorithm which consists of two chains, each of 1.000.000 draws [An, Schorfheide, 2007; Fernandez-Villaverde, 2010; Smets, Wouters, 2003; Rabanal, Rubio-Ramirez, 2005]. To evaluate posterior distributions we use only the last 200.000 draws[4].

In the above-mentioned procedure of estimation which is based on the state-space model, we can divide endogenous variables into two groups: observable and non-observable. The first set consists of: output, consumption, investment and labor. We use data set for a Polish economy from I quarter 1997 to II quarter 2011 which consist of real GDP, real individual consumption expenditure, real gross fixed capital formation and average number of hours worked during the reference week as the proxy of labor[5]. All data were expressed as logs of per capita units, seasonally adjusted and transformed in the percent deviation from steady-state using Hodrick-Prescott filter.

4 Results

Before starting the estimation it is indispensable to define the prior distributions for estimated parameters. The assumptions regarding the prior distribution for estimated parameters are collected in table 1. For all structural parameters we choose beta distribution, because of economic restrictions about possible values of particular parameter. Moreover, as means of distributions we use values that are often used in literature on real business cycle models based on data for postwar US economy[6] [see e.g. Kydland, Prescott, 1982; Hansen, 1985; Hansen, Wright, 1992]. We also assume beta distribution for the autoregressive parameter with mean 0.5 and standard error 0.25[7].

[Table 1.]

The results of the estimation are presented in table 2[8]. The summarized statistics for the posterior distributions prove that all structural parameters, except the discount factor, are identified in estimation[9]. Moreover, they are significantly different from zero. In the case of obtained discount factor posterior is very similar to assumed prior. It is probably caused by very strict prior that was chosen.

[Table 2.]

The mean of posterior for the elasticity of production function is evaluated at the level of 0.59 and it is considerably greater than prior mean. It is also similar to our earlier estimation of the elasticity of production function with respect to capital [Kuchta, Piłat, 2010, p. 28-29] which we use in the calibration exercise. The mean of posterior for the rate of capital deprecation is evaluated at the level of 0.036 and it is greater than assumed prior mean. Moreover, it is also two times greater than value which was found for a Polish economy using annual data on capital stock and its depreciation [Kuchta, Piłat, 2010, p. 28-29]. However both of them are in the 90% posterior interval for this parameter, so they are not significantly different.

The parameter of the autoregressive process for technology is estimated at higher values than prior mean. We obtain a tight distribution with mean 0.83 and the 90% interval of distribution from 0.7 to 0.986. This result proves that technological disturbances are rather persistence in Polish economy. It is also coherent with easier analysis of RBC model in which it is assumed that shocks are highly persistent. However, our estimations may be caused by structure of the model in which we only assume single source of economic disturbance.

Following Adolfson, Laseen, Linde and Villani [2007] and Kolasa [2009] we compare actual series used in estimation with one-step-ahead forecast for observed variables to evaluate fit of the model. Forecasts of observed variables are estimated by applying one-side Kalman filter. The comparison of those two series is summarized in figure 1.

[Figure 1.]

The comparison of forecast series with the observable one proves that estimated model does good job at tracking output. Obviously, the fit for other series is not so good. Predicted series for consumption seems to be quite smooth in comparison with the statistical data. Moreover, average working hours seem to be more volatile than in the sample. However, the overall in-sample fit of the model seems to be acceptable, especially in the light of model's simplicity.

Figure 2 presents selected moments (mean, 5th and 95th percentile of distribution) of the posterior distributions of impulse response functions for the endogenous variables to temporary and positive shock in technology[10].

[Figure 2.]

After positive technological shock, the level of technology rise immediately. In the next periods, it gradually returns to the level of long-run equilibrium. Moreover, response in technology is significantly greater than zero for about two years after shock.

[3] Before estimation, we calibrate the share of consumption and of investment in output at the level of 63% and of 21% respectively. These values were found on the basis of empirical data for Polish economy.

[4] We use Gelman-Brooks statistics to be sure that the MH algorithm will converge.

[5] All data come from the Polish Central Statistical Office.

[6] We do not want to use more reasonable values for a Polish economy, because we would like to check, if there is statistical information on the estimated parameters in the sample that we use in estimation. In other words, we would like to check if there are possible identification problems for the particular parameters.

[7] Chosen priors are rather loose for all parameters except the discount factor. In this case we decided to choose strict prior because of strong connection between this parameter and the steady-state value of real interest rate, which we do not treat in estimation as observable.

[8] The software used is Dynare 4.02.

[9] For more details about issue of identification in the estimation of DSGE models see, among others, [An, Schorfheide, 2007; Canova, Sala 2009].

[10] In the presented results, the temporary technological innovation (ε_t) hit the economy at the end of the second quarter of first year.

Temporary high level of technology causes higher level of production. Moreover, it increases the level of the marginal product of labor and marginal product of physical capital. Changes in marginal productivity of factors encourage individuals to invest and work more than in a steady-state. Investment and hours worked increase immediately after shock and gradually return to their steady-state level. The response in investment is significantly greater than zero for about 2 years after shock. On the contrary, the response in labor effort is considerably greater than zero for about 5 quarters, but after five and a half year we observe that it starts to be significantly lower than zero.

Moreover, the positive shock in technology causes that consumption and capital stock increase gradually after shock. Their hump-shape response is highly persistent, especially in comparison with output. The maximum level of response is observed after about 2 years in case of consumption and after 3 years in case of capital stock. It is also observed that posterior intervals are broad in both cases which suggest rather diffuse posteriors for this response.

5 Results

This paper presented results of estimation of real business cycle model with mechanism of indivisible labor for Polish economy. This model was chosen because of good empirical performance in comparison with other standard RBC models. It was estimated using Bayesian techniques. Our method of estimation was based on the state-space representation of DSGE model. To evaluate the posterior distribution for particular parameters the Metropolis algorithm was used.

Our main results are as follow. Firstly, we obtained that all structural parameters were significant. Moreover, all parameters, except for the discount factor, were identified in estimation. Secondly, obtained parameters were partly comparable with other studies for Polish economy. Thirdly, the overall in-sample-fit was assessed as acceptable, especially taking under consideration model's simplicity. Fourthly, it was obtained that temporary, positive shock in technology increases levels of all endogenous variables. Moreover, only in case of consumption and capital stock it was observed hump-shape and highly persistent responses. Other variables have gradually returned to their steady-state levels after shock.

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Primary Paper Section: A

Secondary Paper Section: AH

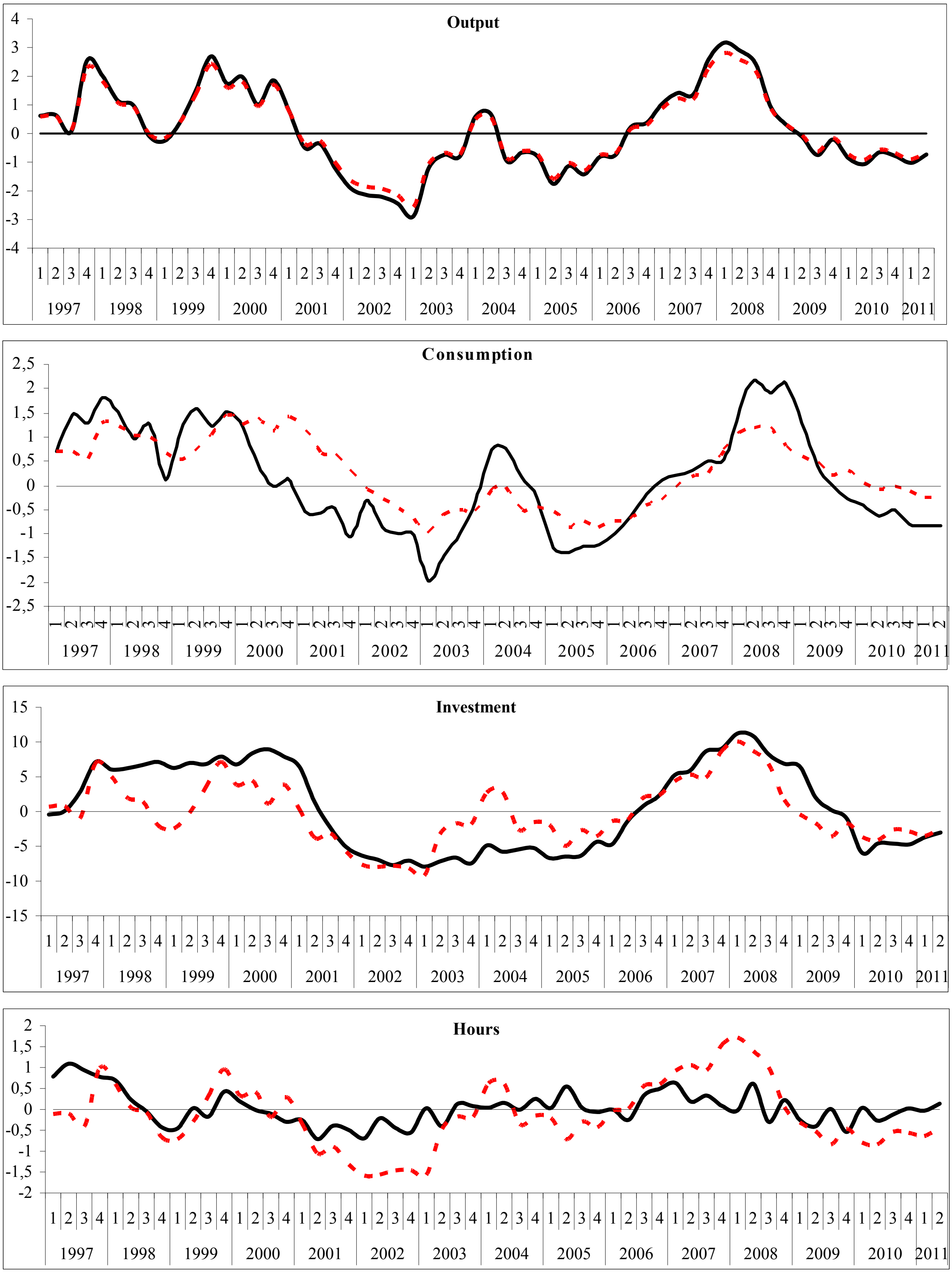
Table 1. Prior distributions for estimated parameters

Parameter	Symbol	Domain	Prior distribution		
			Type	Mean	St. error
Elasticity of production function with respect to capital	α	$(0;1)$	Beta	0.33	0.15
Discount factor	β	$(0;1)$	Beta	0.99	0.005
Rate of deprecation	δ	$(0;1)$	Beta	0.025	0.0125
Autoregressive parameter	ρ	$(0;1)$	Beta	0.50	0.25
Standard error for technological innovation	σ	$(0;\infty)$	Inv gamma	0.03	∞

Table 2. Estimation results: posterior distributions for estimated parameters

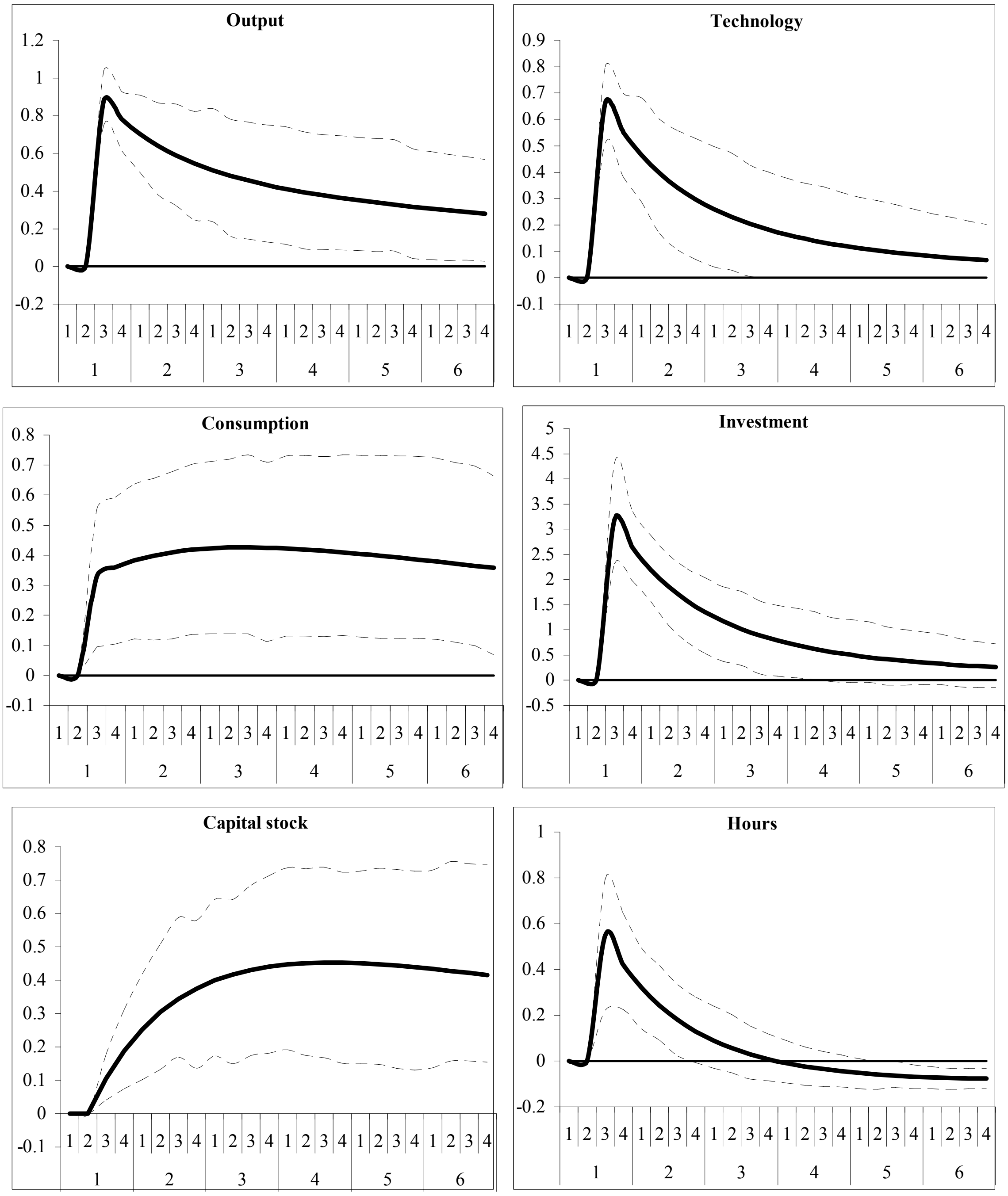
Parameter	Symbol	Posterior distribution			
		Mean	Mode	5%	95%
Elasticity of production function with respect to capital	α	0.594	0.614	0.409	0.780
Discount factor	β	0.990	0.992	0.983	0.997
Rate of deprecation	δ	0.036	0.026	0.009	0.062
Autoregressive parameter	ρ	0.830	0.830	0.707	0.986
Standard error for technological innovation	σ	0.007	0.006	0.005	0.008

Figure 1. Data and one-step-ahead forecasts.



Note: black solid line – data, dashed red line - one-step-ahead forecast.

Figure 2. The posterior distributions of impulse response functions for endogenous variables



Note: solid line – expected value, dashed lines – 5th and 95th percentile.