

## THE PROBLEM OF DETERMINING THE VOLUMES USING MODERN SURVEYING TECHNIQUES

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**Abstract:** The paper deals with the investigation of accuracy of a volume estimation based on the digital elevation model generated from the sets of points with different densities. Investigation of the influence of the point density is provided on the basis of the theoretical bodies, which surface is expressed using a known mathematical function of the planar coordinates, and on the experimental measurements using terrestrial laser scanning and GNSS-RTK method. The results achieved suggest that the density of points significantly affects the accuracy of a volume determination.

**Keywords:** volume, digital elevation model, density of points, terrestrial laser scanning, GNSS-RTK

### 1 Introduction

A calculation of the volumes belongs among the common requirements of the construction and mining industry. To determine a volume we can use different methods and their selection depends on the type and dimensions of an object, character of the border areas, the possibility of the data collection and also on the instrumentation and software.

One of the possibilities of determining a volume between a topographical surface and a reference plane or surface represents a calculation based on the digital elevation model (DEM). However, the accuracy of the DEM based volume calculation depends on the quality of the DEM, which is a function of number variables such as the roughness of the terrain, the interpolation function and interpolation method and the three attributes (accuracy, density and distribution) of the source data [3], [4].

To collect the spatial data needed for a volume calculation, different measurement techniques may be used. In terms of the financial costs and availability, the most commonly used collection method in practice is a conventional surveying method using a total station and the methods based on the GNSS technology. The main disadvantages of these two methods may be seen in a low density of the points, the need to be in a contact with an object and high time consuming. At present, to the fore has also been coming an exploitation of the terrestrial laser scanning (TLS) and digital photogrammetry. Their advantage is the quick and contactless collection of spatial data with a sufficient density. On the other hand, the disadvantage is the need of filtering the unwanted ambient vegetation and objects. The main objective of this paper is to assess the effect of the density of input data on the DEM based volume calculation. To assess this impact, different approaches may be applied. The absolute assessment can be made on the bodies, whose surface is generated using known function of the planar coordinates and their exact volume can be calculated using integral calculus [5], or on the bodies, which exact volume is known. On the other hand, comparing the results obtained from the measurements with different densities of the input data or comparing with the results from different methods, the relative accuracy can be determined [1], [2]. Submitted paper consists of two parts; the first is devoted to analysis on the theoretical surfaces, the second part is focused on the analysis based on the results of the experimental measurements using GNSS-RTK method and TLS.

### 2 Analysis on the theoretical bodies

For this purpose, we defined a rectangular area with the dimensions 20 x 30 m and its origin is given the planar coordinates  $x=0$  m and  $y=0$  m (figure 1). Subsequently, within this area and on its borders, we created the sets of points with a density of 0,02 to 2,00 m (figure 1).

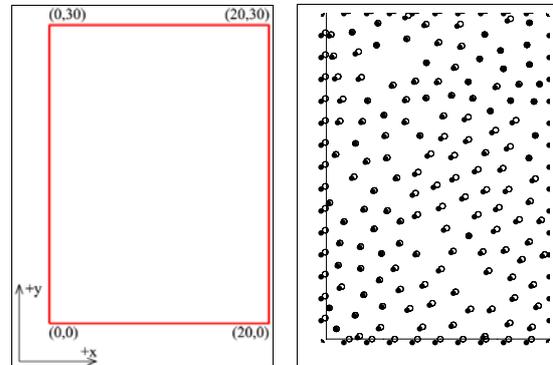


Figure 1: Defined area (left) and a set of points with a density of 2,00 m (right)

In order to create the theoretical surfaces over the defined area, we used two functions for all sets of points:

- theoretical surface A:  $z(x, y) = \sqrt{\frac{x \cdot y}{10}}$ ,
- theoretical surface B:  $z(x, y) = 5 - \left\{ \frac{(x-10)^2}{10^2} + \frac{(y-10)^2}{10^2} \right\}$ ,

where  $x$  and  $y$  are the planar coordinates of points. Based on these sets of the spatial coordinates of points, we finally created the digital elevation model using the Surfer software. For the interpolation we used the Kriging method and the grid size was adjusted according to the density of the source data. Graphical representation of the contoured maps and DEMs of both theoretical surfaces are shown in figures 2 and 3.

Calculation of the volumes from the generated DEMs was also carried out in the Surfer program using the cross-section method. As a reference plane we chose a horizontal plane with the height 0 m. The correct values of the volumes were determined based on the integral calculus, for the theoretical surface A 2065,59 m<sup>3</sup> and for the theoretical surface B 2200,00 m<sup>3</sup>. The results obtained for both theoretical surfaces are shown in table 1. Within the table are also given the values of the relative errors calculated by means of the following formula:

$$r = \frac{V_{IP} - V_{DEM}}{V_{IP}} \cdot 100,$$

where  $r$  is the relative error in [%],  $V_{IP}$  – the volume calculated using the integral calculus,  $V_{DEM}$  – the volume determined by cross-sections from the DEM. Graphical representation of the dependence of the volumes with changing density of the input data are shown in figures 4 and 5.

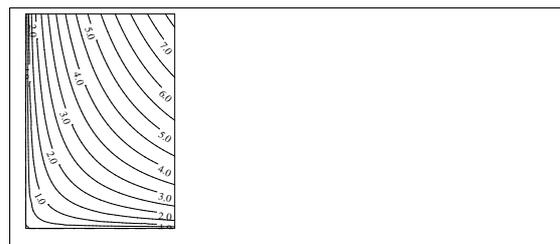


Figure 2: The contoured map and DEM of the surface A

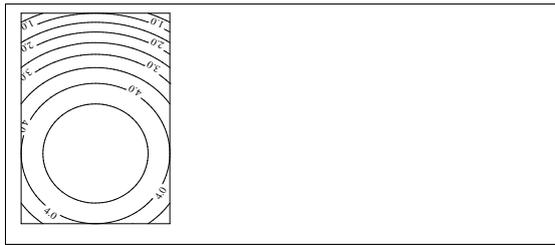


Figure 3: The contoured map and DEM of the surface B

Table 1: Determined volumes and relative errors for the theoretical surfaces A and B

| Theoretical surface A |                   |      | Theoretical surface B |                   |      |
|-----------------------|-------------------|------|-----------------------|-------------------|------|
| Density               | $V_{DEM}$         | $r$  | Density               | $V_{DEM}$         | $r$  |
| [m]                   | [m <sup>3</sup> ] | [%]  | [m]                   | [m <sup>3</sup> ] | [%]  |
| 0,02                  | 2065,12           | 0,02 | 0,02                  | 2199,83           | 0,01 |
| 0,04                  | 2065,12           | 0,02 | 0,04                  | 2199,83           | 0,01 |
| 0,06                  | 2065,09           | 0,02 | 0,06                  | 2199,84           | 0,01 |
| 0,08                  | 2065,04           | 0,03 | 0,08                  | 2199,84           | 0,01 |
| 0,10                  | 2064,95           | 0,03 | 0,10                  | 2199,83           | 0,01 |
| 0,15                  | 2064,70           | 0,04 | 0,15                  | 2199,81           | 0,01 |
| 0,20                  | 2064,37           | 0,06 | 0,20                  | 2199,78           | 0,01 |
| 0,30                  | 2063,53           | 0,10 | 0,30                  | 2199,67           | 0,01 |
| 0,40                  | 2062,54           | 0,15 | 0,40                  | 2199,49           | 0,02 |
| 0,50                  | 2061,43           | 0,20 | 0,50                  | 2199,27           | 0,03 |
| 0,75                  | 2057,87           | 0,37 | 0,75                  | 2198,47           | 0,07 |
| 1,00                  | 2053,97           | 0,56 | 1,00                  | 2197,36           | 0,12 |
| 1,50                  | 2045,10           | 0,99 | 1,50                  | 2194,18           | 0,26 |
| 2,00                  | 2034,65           | 1,50 | 2,00                  | 2190,01           | 0,45 |

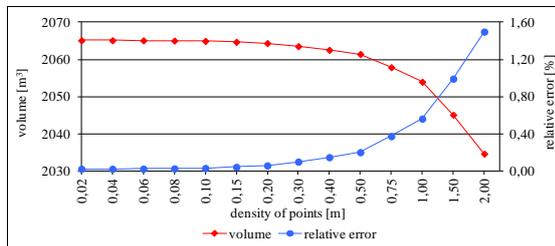


Figure 4: Dependence of the volume on the density of points – theoretical surface A

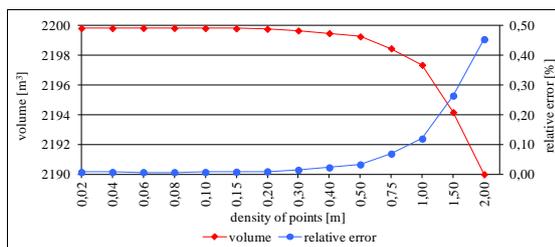


Figure 5: Dependence of the volume on the density of points – theoretical surface B

**3 Analysis on the experimentally measured bodies**

The aim of the experimental measurements was to assess the accuracy of volume estimation on the example of two piles of earth using different collection methods and under different densities of input data. These measurements were carried out on 7<sup>th</sup> of September 2011 on the dumping site at the Apollo bridge in Bratislava using TLS and GNSS-RTK method. For the measurement and subsequent analysis were chosen two newly poured piles of earth in the shape of an irregular truncated cone. The height of both piles of earth was roughly 3 m and the

diameter of a smaller pile was roughly 15 m and 30 m for a bigger pile.

**3.1 Concept of the measurement**

For scanning the piles of earth, we used the Trimble GX scanner (figure 6), which allows scanning to 350 m with speed up to 5000 points per second [7]. We defined the resolution of scanning 20 mm at 50 m and the orientation of the scanner was provided by measuring the planar targets. Scanning of the bigger pile of earth (PE-B) was performed from the 9 scanner stations (5001 – 5009) and to more detailed coverage of the upper part of the pile, 4 stations (5006 – 5009) were placed directly on its upper surface. The smaller pile of earth (PE-S) was scanned together from the 7 stations (5001 – 5003, 5007, 5010 – 5012), from which 2 stations were also on its upper surface (figure 7).



Figure 6: Scanning of the piles at the Apollo bridge in Bratislava

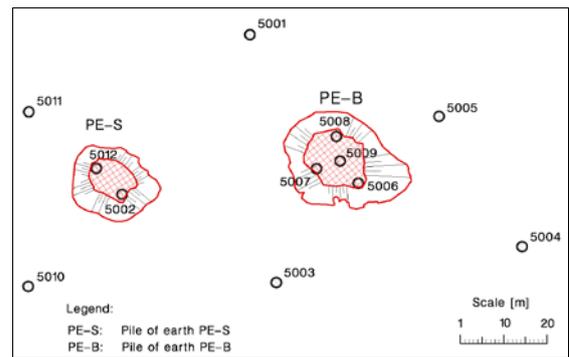


Figure 7: Layout of the scanner stations

Measurement of the piles by means of the GNSS-RTK method was performed using the dual frequency Trimble R6 receiver and the GNSS real-time positioning service SKPOS [6]. To ensure the same 3D reference system for the two data sets collected using GNSS-RTK and TLS, the coordinates and heights of points 5001 – 5012 were determined using GNSS-RTK method as well.

**3.2 Data processing and volume calculation**

Data from TLS were loaded into the Trimble Realworks software for processing. Because the scanner captures everything within its field of view, the point cloud was filtered of the unwanted ambient vegetation and objects (figure 8).

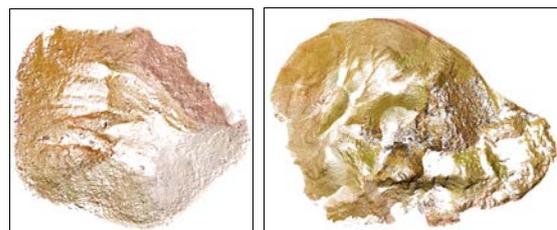


Figure 8: Filtered point clouds of the piles of earth

Thus prepared point clouds were exported into the sets of the spatial coordinates of points with a density from 0,02 to 2,00 m. Creation of the DEMs of the piles (figure 10), needed for calculating the volumes, was performed in the Surfer software environment using the Kriging interpolation method and with a grid size adjusted according to the density of the input data. Similarly, the DEMs from the GNSS-RTK data was also created (figure 11).



Figure 9: Photo of the PE-B

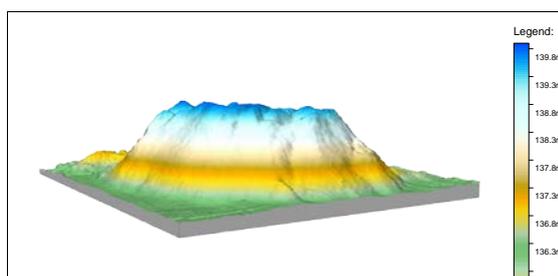


Figure 10: DEM of the PE-B from the TLS data with a density of points 0,10 m

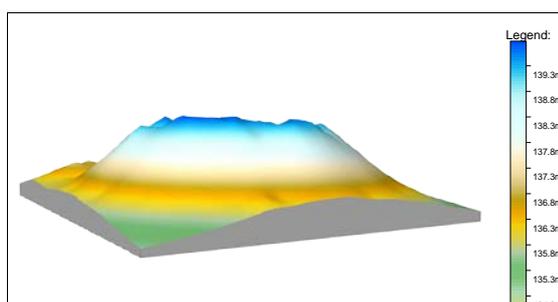


Figure 11: DEM of the PE-B from the GNSS data

Table 2: Determined volumes and relative errors for the piles of earth PE-S and PE-B

| Pile of earth PE-S |                   |              | Pile of earth PE-B |                   |              |
|--------------------|-------------------|--------------|--------------------|-------------------|--------------|
| Density            | $V_{DEM}$         | $r$          | Density            | $V_{DEM}$         | $r$          |
| [m]                | [m <sup>3</sup> ] | [%]          | [m]                | [m <sup>3</sup> ] | [%]          |
| 0,02               | 394,65            | -            | 0,02               | 678,69            | -            |
| 0,04               | 392,99            | 0,42         | 0,04               | 676,56            | 0,31         |
| 0,06               | 391,58            | 0,78         | 0,06               | 672,80            | 0,87         |
| 0,08               | 390,22            | 1,12         | 0,08               | 668,95            | 1,44         |
| 0,10               | 388,59            | 1,54         | 0,10               | 664,88            | 2,03         |
| 0,15               | 385,89            | 2,22         | 0,15               | 661,81            | 2,49         |
| 0,20               | 383,20            | 2,90         | 0,20               | 655,63            | 3,40         |
| 0,30               | 378,08            | 4,20         | 0,30               | 641,51            | 5,48         |
| 0,40               | 373,08            | 5,47         | 0,40               | 635,72            | 6,33         |
| 0,50               | 371,12            | 5,96         | 0,50               | 629,27            | 7,28         |
| 0,75               | 363,83            | 7,81         | 0,75               | 625,69            | 7,81         |
| 1,00               | 358,98            | 9,04         | 1,00               | 617,69            | 8,99         |
| 1,50               | 349,68            | 11,39        | 1,50               | 602,49            | 11,23        |
| 2,00               | 347,68            | 11,90        | 2,00               | 593,87            | 12,50        |
| <b>GNSS</b>        | <b>348,28</b>     | <b>11,75</b> | <b>GNSS</b>        | <b>600,31</b>     | <b>11,55</b> |

The resulting DEMs of the piles of earth were used to calculate the volumes using the cross-section method in the Surfer software. As a reference plane we used a horizontal plane with the height 136,60 m for the PE-S and 136,30 m for the PE-B. The results obtained for the different densities of input data for both piles are shown in table 2. The table also contains the values of volumes obtained from the GNSS-RTK data created DEMs. The relative errors mentioned in table 2 are related to a volume determined from a set of points with 0,02 m density. Graphical presentation of the results is given in figures 12 and 13.

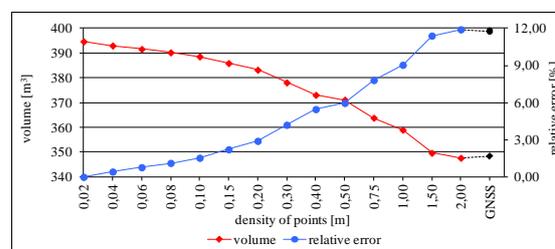


Figure 12: Dependence of the volume on the density of points – PE-S

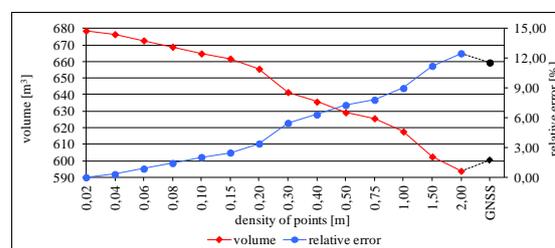


Figure 13: Dependence of the volume on the density of points – PE-B

#### 4 Conclusion

This article was focused on assessing the accuracy of a volume determination from the point of view of a varying density of the input data and employed data collection method as well. The analysis was performed on the two mathematically defined surfaces and on the basis of the measurements realized using TLS and GNSS-RTK method.

In the case of the bodies, whose surfaces were defined by means of a mathematical function, the relative errors acquires maximum values 1,50% and 0,45% at the 2 m density of input data. On the other hand, in the case of the experimentally measured piles of earth, the relative error already overlays value 1% at the density of points 0,08 m and the maximum errors acquires values up to 11,90% for the PE-S and 12,50% for the PE-B. From a comparison of the volumes obtained from the TLS and GNSS-RTK data we can conclude that the relative errors acquires values roughly 11,50% and we assume that this result is due to an insufficient coverage of the surfaces of the piles of earth.

From the above mentioned results we can conclude that the density of points has a significant influence on the DEM based volume calculation. The density of the points is reflected in the approximation of a real surface, and therefore its influence increases with the roughness and complexity of the area of interest.

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