REVENUE EFFICIENCY IN THE CZECH REPUBLIC AND SLOVAKIA

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Abstract: This paper applies the DEA to evaluate revenue efficiency using the traditional (TRE) and new (NRE) model. We show how to make the relevant calculations in RStudio and discuss application in the banking area. Using balanced panel data from 2008 to 2017, we compared the efficiency of Czech and Slovak banks. The results showed that the TRE ranged from 24.15% to 100%, and NRE ranged from 29.68% to 100%. The results showed that a higher efficiency was in the case of banks located in Czech Republic and pointed to the fact that large banks were the most efficient. The results of nonparametric test for equality of densities pointed to the fact that it depends on selected methodology, bank size and also the location of banks. The results were also verified by the GMM model, which signalized, that the level of efficiency depended mainly on the efficiency in the previous year, but also on the location of the bank in the new revenue model.

Keywords: Bank, Czech Republic, Data Envelopment Analysis, Distributional approach, Generalised Method of Moments, Revenue efficiency, Slovakia, RStudio.

1 Introduction

Commercial banks represent the largest segment of the Czech and Slovak financial system. Both countries could be considered as representatives of the bank-oriented financial system, where the commercial banks play a crucial role. According to data published by the National bank of Slovakia and Czech national bank, we can say that the share of commercial banks assets on the total assets of all financial intermediaries is higher than 60%. There is, therefore necessary, to study efficiency of these institutions in order to differentiate the efficient one from non-efficient ones and to find out which conditions are the most prerequisite to becoming efficient. When measuring efficiency based on the non-parametric DEA method, there are two different situations: one with common unit prices of outputs for all Decision-Making Units (DMUs) and the other with different prices of outputs from unit to unit. The concept of revenue efficiency was first introduced by Farrell (1957) and then developed by Färe et al. (1985). In this model, they assumed that output prices are the same across all units. However, the common price and revenue assumption is not always valid in real business, and it has been shown that efficiency measures based on this assumption can be misleading. So a new revenue efficiency model was introduced by Tone (2002). In our paper we want to compare the results obtained with the traditional and new revenue model. We want to present the usage of these methods in efficiency evaluation in software RStudio, namely through the application of package "Benchmarking".

This study aims to examine the revenue efficiency of the Czech and Slovak banks using the DEA and try to answer the research question whether the efficiency differs across banks with different size, and banks from different countries. The efficiency is evaluated in a sample of 14 banks during the period from 2008 to 2017 based on the unconsolidated data published within annual reports of analysed banks. In the next step, the estimated efficiencies are tested for equality of densities of two given distributions via the test prepared by Li et al. (2009). Then the system generalised method of moments (GMM), is applied to analyse if the efficiency in the previous year, location and bank size can be considered as variables with significant impact on bank efficiency. This study contributes to the existing literature by comparing of traditional and new revenue model and by examining the statistical differences in equality of efficiencies in terms of the size and location. One of the benefits of the paper is filling the current gap in the scientific literature, as this type of analysis is missing in the literature concerning the banking sectors in the Czech Republic and Slovakia.

To fulfil the aim of the paper, the structure of the paper is as follow. The review of the relevant literature is described in the second section. The third section describes the methods. Next, the fourth section presents the data and results. In last section, we conclude the paper.

2 Literature review

The literature on bank efficiency has expanded during the last years, where the researchers started to prefer parametric and nonparametric methods to evaluate the revenue efficiency instead of ratio analysis. As mentioned by Dong et al. (2014), since both parametric and non-parametric approaches have their advantages and limitations and since the actual level of revenue efficiency is unknown, the choice of an appropriate efficiency estimation approach has been quite controversial. Therefore, in the banking area, some researches, for example, Rossi et al. (2005), Olson & Zoubi (2011), or Ghroubi & Abaoub (2016), prefer parametric method, while some researches like Havranek & Irsova (2013), Pancurova & Lyócsa (2013), Siranova & Cupic (2015), Prior Jiménez et al. (2016), Gavurova et al. (2017), Černohorská et al. (2017), Cupic & Siranova (2018), Phang & Raweewan (2018), or Palečková (2019), mainly used the non-parametric approach. We can also find several studies, for example, Delis et al. (2009), Irsova (2009), Tan (2016), Dinh et al. (2019) or Ruinan (2019) comparing the results of revenue efficiency estimated by both methods simultaneously. Most of these studies apply the traditional revenue efficiency frontier. However, in the study of Pancurova & Lyócsa (2013), we can also find the application of a new revenue efficiency function.

Havranek & Irsova (2013) analysed what drove bank cost and revenue efficiency in the transition countries of Central Europa and compare results with those for the United States. They applied the traditional DEA model to evaluate efficiency for the period of 1995-2006, based on intermediation approach. They found out that the largest banks were most revenue efficient. Also, foreign banks reported higher cost and revenue efficiency. The revenue efficiency was higher than cost efficiency, which signalised that banks were more successful in gaining profits on average.

Pancurova & Lyócsa (2013) estimated efficiencies and their determinants for a sample of eleven Central and Eastern European Countries over the 2005-2008 period. They estimated cost and revenue efficiency using new DEA models. Within the second stage, they tested the separability assumption and estimated determinants of efficiency by using a truncated regression model. They found out that the size and financial capitalisation of banks were positively associated with cost and revenue efficiency, while the loans-to-assets ratio was negatively associated with cost efficiency but positively associated with revenue efficiency. Moreover, foreign banks were more cost-efficient but less revenue efficient than domestic banks, suggesting different banking behaviour between domestic and foreign banks.

The impact of concentration on bank profitability was analysed in the study of Černohorský (2015) who used correlation and regression analysis in the Czech banking sector during the period 2003-2012. He found out that there existed an inverse relationship between the degree of concentration and the size of the profits of the banking sector, which is not the standard output of previous studies.

Siranova & Cupic (2015) investigated the influence of Slovakia accession to the European Union on the efficiency of Slovak banks over the period 2001-2009. They found out a significant association between accession and bank efficiency. Also, the found small banks were more efficient than large banks, which was partly because large banks were oversised and operated at the decreasing returns to scale. They also analysed the impact of accession into the Euro area using data for the period 2000-2013. Cupic & Siranova (2018) suggested that the efficiency of Slovak banks was not affected by macroeconomic conditions and banking reforms, which was in line with the argument that Slovak banking sector was in the advanced stage of development when the influence of these factors was of less importance. During this period, large banks were more efficient than small banks, and foreign banks were more efficient than domestic ones.

Prior Jiménez et al. (2016) focused on the Spanish banking sector for the 2005-2009 period. They applied traditional DEA method to evaluate cost and revenue efficiency of Spanish commercial banks, savings banks and credit unions during both the pre-crisis years (from 2005 to 2007) and crisis years (2008 and 2009). They applied Li et al. (2009) test to analyse differences between groups of banks according to the type of bank and crisis years. They found out that differences did not exist when comparing saving banks and credit unions. In contrast, commercial banks were more efficient than the other two bank types.

Gavurova et al. (2017) examined the cost and revenue efficiency of banking sectors within the European Union countries over the period 2008-2015. After the estimation of efficiencies, they tested if there exist differences in term of size, European region and crisis years. They found out that the results differ in the specified group, so they concluded that the results of analysis depended on size, location and crisis period.

Phang & Raweewan (2018) examined the cost, revenue and profit efficiency of the Cambodian banking system over the period 2010-2013 by traditional DEA model. They found out that large banks were more cost, revenue and profit efficient than smaller counterparts. Moreover, foreign banks, on average, were more cost, revenue and profit efficient than domestic banks. Results suggested that the banking sector still had the potential for cost savings as well as revenue and profit increases.

Ruinan (2019) applied both SFA and traditional DEA to evaluate cost and profit efficiency of the largest banks in the United States and Canada for the 2008-2017 period. The results regarding cost and profit efficiency confirmed the prior studies indicating a low correlation between these two measures. However, SFA and DEA produced very different and uncorrelated results, though DEA generated overall lower efficiencies. The findings suggested that methodology cross-checking, along with information regarding variables selection, are necessary before decision making.

The determinants of bank efficiency were also evaluated by Palečková (2019), who found out that larger banks with higher liquidity risk and with the lower value of the net interest margin were more efficient than smaller ones. According to the results, we can say that banks were highly efficient during the economic expansion with a lower value of the inflation rate.

The last mentioned, Dinh et al. (2019) evaluated efficiency of 30 commercial banks in Vietnam using both parametric (SFA) and non-parametric (DEA) approach during the period of 2011–2015. They applied Tobit regression model, to investigate the impact of bank size, bank age, and the ownership feature on the efficiency. They found out that there is small level of similarity in efficiency rankings identified from the SFA and DEA models. In terms of efficiency determinants, the results showed that all three variables of size, age, and state ownership have a positive impact on bank efficiency.

3 Methodology

Charnes et al. (1978) first developed the Data Envelopment Analysis (DEA) under the constant returns to scale assumption and this way provided a measure of technical efficiency. Following Farrell (1957), and Färe et al. (1985), a sequence of linear programmes was applied to construct revenue efficiency frontiers, which is now used to measure traditional revenue efficiency. Berger & Mester (1997) argue that revenue efficiency measures the change in a bank´s revenues adjusted for a random error, relative to the estimated revenues obtained from producing an output as efficient as the best practice bank. According to Tan (2016), revenue efficiency emphasises the fact that the banking operations aim to maximise revenues. The traditional revenue efficiency model (TRE) assumes that the unit price of outputs is identical among DMUs. According to the Pancurova & Lyócsa (2013), to be revenue efficient, the DMU must be both technically efficient (adopting the best practice technology) and allocative efficient (selecting the optimal mix of outputs to maximise the revenues for a given input).

We define $\mathbf{y_o}$ as the $s \times 1$ vector of the o-th production unit´s s outputs $(r = 1,...,s)$, $\mathbf{x_o}$ is the $m \times 1$ vector of its m inputs $(i = 1,...,m)$, $\mathbf{Y}$ is the $s \times n$ matrix of outputs (n denotes the number of DMUs, $(j = 1,...,n)$, and $\mathbf{X}$ is the $m \times n$ matrix of inputs. Let us consider we have prices associated with outputs. Let $\mathbf{p} = (p_1,..., p_s)$ be the common unit output-price or unit-revenue vector. Then the revenue efficiency ρ of DMU_o is defined as the ratio between the actual revenues and maximal revenues. Where maximal revenues are calculated through the optimal solution of the constant returns to scale revenue maximisation model. For detail description of model see Cooper et al. (2007).

In traditional revenue efficiency DEA models, we assume that output prices are the same across all decision-making units. However, actual markets do not necessarily function under perfect competition, and unit output prices might not be identical across all DMUs. As pointed out by Tone (2002), the traditional DEA revenue efficiency model does not take account of the fact that revenues can be increased by increasing the output factor prices. Under the traditional DEA model, the revenue function is homogeneous of degree one in output prices, and the scaling factor cancels out in the revenue efficiency ratio, and thus, the two DMU will be assigned the same measure of revenue efficiency irrespective of the fact that they have significantly different output prices. It represents a severe drawback for assessing relative efficiency levels under the traditional DEA model an is caused by the peculiar structure of the DEA model which exclusively focuses on the technical efficiency of two DMU and cannot take account of variations in unit output prices between the DMUs. Therefore, in order to avoid this shortcoming, Tone (2002) proposed a new scheme for evaluating revenue efficiency under which the production technology is homogeneous of degree one in the total revenues as distinct from being homogeneous of degree one in the output prices under the traditional DEA model. As mentioned by Dong et al. (2014), it means that under the new DEA model DMUs with different output prices will return different measures of revenue efficiency.

The new revenue efficiency model (NRE) is based on the definition of another revenue-based production possibility set P_R as $P_R = \{(x, \bar{y}) | x \geq X\lambda, \bar{y} \leq \bar{Y}\lambda, \lambda \geq 0\}$. Where $\bar{Y} = (\bar{y}_1,..., \bar{y}_n)$ with $\bar{y}_j = (p_{1j}y_{1j},..., p_{sj}y_{sj})$. Here we assume that the matrices Y and P are non-negative, and elements of $\bar{y}_{rj} = (p_{rj}y_{rj})(\forall(r, j))$, are denominated in homogeneous units in monetary terms (e.g. euro). According to Cooper et al. (2007), the new revenue efficiency $\bar{\rho}_o$ is defined:

$$\bar{\rho}_o = \frac{e\bar{y}_o}{e\bar{y}_o^*} \tag{1}$$

Where $e \in R^m$ is a row vector with elements being equal to 1, and $\bar{y}_o^*$ is the optimal solution for the linear programmes given below:

New Revenues

$$e\bar{y}_o^* = \max_{\bar{y}, \lambda} e\bar{y} \tag{2}$$

Subject to $x_o \geq X\lambda\,;\ \bar{y} \leq \bar{Y}\lambda\,;\ \lambda \geq 0$ (3)

In the new revenue efficiency model the optimal output mix $\bar{y}_o^*$ that uses the input x_o can be found independently of the DMU´s current unit price p_o, whereas in the traditional revenue efficiency model keeping the unit revenue of DMU *j* fixed at p_o when we was searching for optimal output mix y^*. These are fundamental differences between the two models. Using traditional revenue efficiency model, we can fail to precisely the existence of other more profitable output mixes. We can demonstrate a simple example involving three DMUs A, B and C with each using one input (x) to produce two outputs (y_1, y_2) along with output prices (p_1, p_2). The data and the resulting measurement are exhibited in Tab. 1. For DMUs A, B and C, the traditional revenue model gives the same efficiency score $\rho^* = 1$. As they used the same amount of input to produce the same amount of outputs, we can consider them as technical efficient. The traditional revenue model assumes that the unit price of outputs is identical among, so do not take into account the actual prices of production units.

Tab. 1: Comparison of traditional and new revenue efficiency

	DMU A	DMU B	DMU C
x	1	1	1
y_1	10	10	10
y_2	10	10	10
p_1	10	1	1
p_2	10	1	10
$\bar{y}_1$	100	10	10
$\bar{y}_2$	100	10	100
$\bar{y} = \bar{y}_1 + \bar{y}_2$	200	20	110
e_1	1	1	1
e_2	1	1	1
Traditional revenue model ρ^*	1.00	1.00	1.00
New revenue model $\bar{\rho}^*$	1.00	0.10	0.55

Source: Prepared by authors according to Cooper et al. (2007)

The new scheme devised as in Tone (2002) distinguishes DMU A from DMU B and DMU C by according them different revenue efficiency scores. This is due to the difference in their unit revenues. Moreover, DMU A is judged as revenue efficient. We can also see the drop in DMU C from 1 to 0.55 and in the case of DMU B from 1 to 0.10. We can explain the drop in DMU B´s and DMU C´s performance by their lower revenue structure. We can see that DMU B produces ten units of output 1 with a price of 1 price unit per one unit and ten units of output 2 with a price of 1 price unit per one unit. It means that by using the same amount of input, DMU B can generate a lower total revenue equal to 20 price units. In case of DMU C, we can see that it produces ten units of output 1 with a price of 1 price unit per one unit and ten units of output 2 with a price of 10 price unit per one unit, which generates total revenues equal to 110 price units. The last one, DMU A produces ten units of output 1 with a price of 10 price unit per one unit and ten units of output 2 with a price of 10 price units per one unit, which generates total revenues equal to 200 price units. It indicates, that all DMUs use the same amount of input to produce the same amount of outputs, but when we take into account different unit prices, we can see, that the total revenues of production units are different, therefore we could not consider them as the same revenue efficient. When we would like to consider DMU B and DMU C also as efficient, it is necessary to increase their revenues to the optimal level of 200 price units, the same as it is in the case of DMU A. There are two main ways how to obtain the optimal revenues. The first one is to increase the prices at the same level as it is in the case of DMU A. It means that the DMUs will produce the same amount of outputs at the same level of their prices by using the same level of inputs. However, in this situation, the DMUs could not get a competitive advantage in the form of prices. The second way is to increase the level of the produced outputs. In the case of DMU B, to obtain the optimal revenues, it is necessary to increase the production of first output from 10 to 100 units, and the production of second output also from 10 to 100 units. In the case of DMU C, it should increase the production of the first output from 10 to 100 units at the unchanged level of second output. This way, the production units will be able to obtain the same level of optimal revenues without changing the level of their prices. As we suppose that on the market the consumers prefer products with lower prices, we can suppose that the DMU B with the lowest prices can get a competitive advantage, gain more consumers, and this way by selling the higher amount of products can get the same level of optimal revenues.

Having estimated revenue and new revenue efficiency, we will test distributions of efficiencies for two sub-groups. Li et al., (2009) proposed a nonparametric test for equality of multivariate densities, comprised of continuous and categorical data. Let *X* and *Y* be multivariate vectors of dimension $q + r$, where *q* denotes the number of variables from the first sample, and *r* denotes the number of variables from the second sample. According to Racine (2012) test statistic can be constructed based on the integrated squared density difference given by

$$I = \int \left[f(x) - g(x) \right]^2 dx = \int \left[f(x) dF(x) + g(x) dG(x) - f(x) dG(x) - g(x) dF(x) \right] \quad (4)$$

where $F(\cdot)$ and $G(\cdot)$ are the cumulative distribution functions for *X* and *Y*, respectively, and where $\int dx = \sum_{x^d \in S^d} \int dx^c$. Replacing the first occurrences of $f(\cdot)$ and $g(\cdot)$ by their leave-one-out kernel estimates, and replacing $F(\cdot)$ and $G(\cdot)$ by their empirical distribution functions, we obtain the test statistics (for more detail see Li et al. (2009) and Racine (2012)). Based on this methodology and based on the paper by presented by Prior Jiménez et al. (2016), we used the nonparametric test for equality of densities to test whether two given distributions, say $f(\cdot)$ and $g(\cdot)$, estimated nonparametrically via kernel smoothing, differ statistically in terms of the bank size, and in terms of location. Based on this methodology and based on the paper presented by Prior Jiménez et al. (2016), we used the nonparametric test for equality of densities to test whether two given distributions estimated nonparametrically via kernel smoothing, differ statistically.

To verify our findings, in the last step we apply the system generalised method of moments (GMM), which is appropriate to deal with explanatory variables that are not strictly exogenous. We involve dummy variables into the model, as we want to analyse if the location and bank size can be considered as variables with a significant impact on bank efficiency. The model is defined as follows:

$$y_{i,c,t} = \alpha y_{i,c,t-1} + \beta DummyLocation_{i,c,t} + \gamma DummySize_{i,c,t} + \varepsilon_{i,c,t} \quad (5)$$

Where $y_{i,c,t}$ is the revenue efficiency of the bank *i* in the country *c* at the time *t* measured via traditional or new revenue model; $y_{i,c,t-1}$ is the revenue efficiency of the bank *i* in the country *c* at the time *t*-1; $DummyLocation_{i,c,t}$ is a dummy for the country where bank *i* is located at the time *t*, and $DummySize_{i,c,t}$ is the dummy for the bank size.

3 Results and discussion

We will illustrate using revenue DEA models under the assumption of a variable returns to scale, to measure the revenue and new revenue efficiency of commercial banks in the Czech Republic and Slovakia. We assume seven commercial banks located in the Czech Republic and seven commercial banks located in Slovakia. In our research, we have focused only on the evaluation of domestic commercial banks; the foreign-controlled branches were not involved. The analysis is based on the data of domestic banks, which comprises more than 77% of total banking assets in 2017 in the case of Slovakia, and more than 70% of total banking assets in 2017 in the case of Czech Republic. The data are taken for the period from 2008 to 2017. All data were reported in EUR as the reference currency. The data in the national currency (Česká koruna – CZK), were converted by using the official exchange rate of the Czech National Bank at the end of the specified year. The source of the unconsolidated data is annual reports of analysed banks. Based on the literature review, we adopt the intermediation approach for selecting inputs and outputs of banks. The intermediation approach is the primary approach for modelling of banking activity, focusing in particular on the role of banks as financial intermediaries between depositors and end-users of bank assets. As mentioned by Sealey Jr & Lindley (1977) deposits and other liabilities, together with real resources are treated as inputs, whereas outputs include only bank assets that generate revenues. We consider three inputs: The number of full-time employees (x_1), Total fixed assets in thousands of EUR (x_2), and Total deposits in thousands of EUR (x_3). We include two outputs: Total loans in thousands of EUR (y_1), and Total other earning assets thousands of EUR (y_2). Let denotes p_1 the price of output y_1, and p_2 the price of output y_2. The price of loans (p_1) in EUR can be calculated as the ratio of total interest income to total loans, and the price of other earning assets (p_2) in EUR can be calculated as the ratio of non-interest income to total other earning assets.

Practical calculation of revenue and new revenue efficiency is realised using the software RStudio. RStudio is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS and can be downloaded on the web page: https://rstudio.com/. RStudio is very much a vehicle for newly developing methods of interactive data analysis. It is developing fast and has been extended by an extensive collection of *packages*. However, most programs written in R are principally ephemeral, written for a single piece of data analysis. (www.r-project.org). One of the packages is a package „Benchmarking", prepared by Bogetoft et al. (2018). Bogetoft & Otto (2010) in their work used software R for calculating efficiency not only for DEA models but also for SFA models. The package „Benchmarking" contains methods to estimate technologies and measure efficiency using DEA while supporting different technology assumptions (Free disposability hull, Variable returns to scale, Constant returns to scale, Decreasing returns to scale, Increasing returns to scale), and using different efficiency measures (Input based, Output based, Hyperbolic graph, Additive efficiency, Super efficiency, Directional approach). The methods can solve not only standard models, but also many other model variants, and they can be modified to solve new models. As we want to eliminate the frontier shift effect, the data are evaluated within one sample, not separately for each year. This way, the efficiency frontier is estimated from all observed values.

To solve the traditional revenue maximisation problem using RStudio, we first load the data from MS Excel file, where the file is prepared for each year separately (*data.xls*). The next step requires to install the package Benchmarking. In the next step, we have to apply the revenue maximisation DEA model, using the procedure *revenuet.opt* from the Benchmarking package. This command estimates the optimal output vector (*yopti*) that maximises revenues in the context of a DEA technology. The part of the command is to define which variables will act as inputs (the matrix of inputs, *x*), outputs (the matrix of outputs, *y*), output prices (as a matrix, *p*) and used technology (variable returns to scale "*vrs*") of the applied model. To calculate traditional revenue efficiency, we have to find the actual revenues ($robs = \sum_{r=1}^{s} p_{rq} \cdot y_{rq}$) and the optimal revenues ($ropt = \sum_{r=1}^{s} p_{rq} \cdot y_{rq}^{*}$), and then we divide the actual revenues by the optimal revenues. It is calculated in R by using the inner product *%*%*, or matrix multiplication where the function *t* is matrix transposed. For calculation of traditional revenue efficiency, it is necessary to select data from diagonals of formed matrices (*pobs1*; *popt1*). By dividing these values, we obtain traditional revenue efficiency (*tre*) of evaluated banks. The process of calculation of traditional revenue efficiency by using RStudio can be entered via the following commands:

```
data = read_excel("C:/……/data.xlsx")
library("Benchmarking")
x = with(data, cbind(x1,x2,x3))
y = with(data, cbind(y1,y2))
p = with(data, cbind(p1,p2))
yopti = revenue.opt(x,y,p, RTS="vrs")
robs = y %*% t(p)
ropt = yopti$yopt %*% t(p)
ropt1 = diag(ropt)
robs1 = diag(robs)
tre = robs1/ropt1
print(cbind("tre"=c(tre)), digits=4)
```

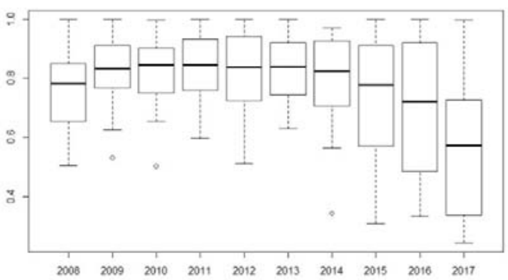
By multiplying the total loans (y_1) and a total other earning assets (y_2) with their respective unit revenues (p_1, p_2), we obtain new output data set (*yn*) which can be used to calculate new revenue efficiency. By switching original output prices directly to the new output, the price data does not enter the new model but is replaced by the row vector with elements being equal to 1 (e_1, e_2). This new data set is used in the process of revenue maximisation DEA model, using the procedure *revenue.opt*. To calculate new revenue efficiency, we also have to find the actual revenues (*robsn*) and the optimal revenues (*roptn*), and then we divide the actual revenues by the optimal revenues. By dividing values from the diagonals of matrices (*robs1n*; *ropt1n*), we obtain new revenue efficiency (*nre*) of evaluated banks. The process of calculation of new revenue efficiency by using R can be entered via the following commands:

```
x = with(data, cbind(x1,x2,x3))
yn = with(data, cbind(y1*p1,y2*p2))
pn = with(data, cbind(e1,e2))
ynoptin = revenue.opt(x,yn,pn, RTS="vrs")
robsn = yn %*% t(pn)
roptn = ynoptin$yopt %*% t(pn)
ropt1n = diag(roptn)
robs1n = diag(robsn)
nre = robs1n/ropt1n
print(cbind("nre"=c(nre)), digits=4)
```

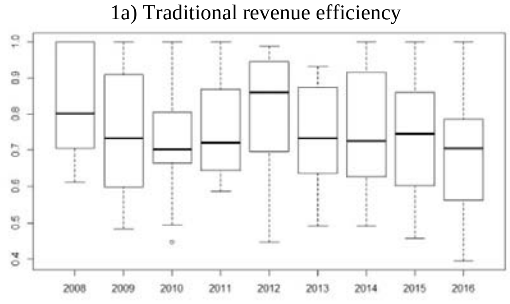
According to the methodology described in the previous section, we evaluate the revenue and new revenue efficiency of Czech and Slovak banks. The results are presented in the following order. First, we report the estimates of overall revenue and new revenue efficiency during 2008-2017. Next, we use univariate cross-tabulation to trace revenue and new revenue efficiency under the alternative classification based on different parameters like bank size and location. As mentioned by Ray & Das (2010), the univariate approach does not satisfactorily analyse the distributional structure of the efficiency estimates. To analyse this aspect, the entire distribution of efficiencies based on kernel densities under various conditioning schemes is also presented.

Figure 1 presents the results of revenue and new revenue model. When we look at the results of the traditional revenue model, we can see a decline during the last years of the analysed period. The median values were approximately at the same level during the period from 2008 to 2015. As we can see under this approach the most inefficient banks should be considered as outliers within the sample in 2009, 2010 and 2014, as most of the efficiencies were located within the interval of approximately 0.6-1. The minimum average value was reached in 2017, the maximum average value in 2013. The average revenue efficiency at the beginning of the analysed period was 77.38% indicating that on average, banks could increase their revenues by 22.62% by producing outputs in optimal combination under a given level of inputs. At the end of the analysed period, the average revenue efficiency was 56.64%, indicating potential revenue increase equal to 43.36%. Under the new revenue model, the outlier can be seen only in 2010. We can see a decline until 2010, with a slight increase between 2010 and 2012, which was replaced by a decreased since 2012. The minimum average value was reached in 2017, the maximum average value in 2008. The average new revenue efficiency at the beginning of the analysed period was 82.41% indicating that on average, banks could increase their revenues by 17.59% by producing outputs in optimal combination under while maintaining the given output prices and with a given level of inputs. At the end of the analysed period, the average new revenue efficiency was 63.11%, indicating potential revenue increase equal to 36.89%. Generally, based on the results displayed in Figure 1, we can say that analysed banks were more efficient under the new revenue model than under the traditional revenue model. As the reason of decline in revenue efficiency can be considered the decline in interest rates set up by central banks in both countries. This decline was pass-through into the interest rates on the interbank market, and also into the interest rates connected on bank products like bank loans and bank deposits. Due to the decline in interest rates for loans, banks were not able to generate such a high value of interest income as at the beginning of the analysed period. While at the beginning of the analysed period the commercial banks were able to generate interest income with an average value of 509604 thousand of EUR at the end of the analysed period it was only 389686 thousand of EUR. As we know, the commercial banks try to replace this loss of interest income by decreasing operating expenses or by increasing non-interest income to prevent a decline in their profitability. Therefore, in case of non-interest income, we can see an increase during the analysed period. While at the beginning of the analysed period the commercial banks were able to generate interest income with an average value of 115932 thousand of EUR at the end of the analysed period it was only 160239 thousand of EUR. We could see that the banks were able to increase their non-interest income, but the increase was not so high compared to the decline in interest income. Therefore, we can suppose that this fact had a negative impact on banks revenue efficiencies during the last years of the analysed period.

Figure 1: Revenue efficiencies given by DEA models according to years



1a) Traditional revenue efficiency



1b) New revenue efficiency

Source: Prepared by authors

In the univariate approach, the estimates efficiency scores are analysed by a single attribute. Tab. 2 presents the results according to the location. Under both approaches, Czech banks appeared as the more efficient as the average efficiencies of Czech banks were higher than Slovak ones. On the other hand, Slovak banks reached lower average efficiencies. A higher decline in interest rates can explain the generally lower efficiency of Slovak banks compared to the Czech banking sector. As it was mentioned this decline had a negative impact on generating interest income of commercial banks. The development of the average traditional revenue efficiency in two groups of banks indicates a decline in the efficiency in the case of Slovak banks, while the efficiency in case of Czech banks was relatively stable till 2016 and then decrease can be seen in last year of the analysed period. The similar development could be also seen in the case of new revenue model, but decline in last year in the case of Czech banks is not so significant.

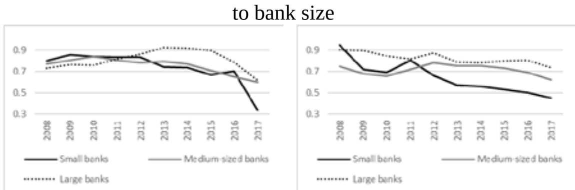
Tab. 2: Location and revenue efficiencies given by DEA models

	Traditional revenue model ρ^*		New revenue model $\overline{\rho}^*$	
Country (No. of banks) / Year	Czech Republic (N = 7)	Slovakia (N = 7)	Czech Republic (N = 7)	Slovakia (N = 7)
2008	0.8340	0.7136	0.8712	0.7769
2009	0.8270	0.8111	0.8750	0.6268
2010	0.8485	0.8012	0.7954	0.6393
2011	0.8604	0.7785	0.7951	0.7272
2012	0.8505	0.7880	0.9139	0.6766
2013	0.9112	0.7425	0.8350	0.6432
2014	0.9112	0.6722	0.8397	0.6314
2015	0.8356	0.6357	0.8528	0.5925
2016	0.8231	0.5776	0.9393	0.5555
2017	0.6336	0.4992	0.7677	0.4945
2008-2017	0.8335	0.7020	0.8385	0.6364

Source: Prepared by authors

The relationship between efficiency and size of banks is presented in Figure 2. The analysed banks were divided into three groups: small banks, medium-sized banks and large banks. In terms of absolute amounts, the threshold is defined based on the total assets of the analysed banks during the whole analysed period. Within the group of small-sized banks, there are banks with assets less than 25th percentile of assets of all analysed banks. Within the group of medium-sized banks, there are banks with assets between 25th and 75th percentile of assets of all analysed banks. Moreover, within the group of large-sized banks, there are banks with assets higher than 75th percentile of assets of all analysed banks. The results indicate that large banks were more efficient than small and medium-sized banks. In past years the last efficient were small banks under both approaches. In the case of large banks, the minimum average value of traditional revenue model was reached in 2017, the maximum average value in 2013. In the case of the medium-sized banks, the minimum was also reached in last year and a maximum in 2010. In the case of the small banks, the minimum was reached in 2017 and a maximum in 2009. In the case of a new revenue model, the decline tendency can be seen in all types of banks with the minimum values in the last years.

Figure 2: Revenue efficiencies given by DEA models according to bank size



2a) Traditional revenue efficiency 2b) New revenue efficiency

Source: Prepared by authors

The presented results pointed to the differences between banks located in different countries, and also with different size. Therefore, we now turn to the analysis of the distribution of revenue and new revenue efficiency. We apply the test presented by Li et al. (2009), to compare if there exist significant differences between both approaches, and also between different size groups and groups according to location.

The comparative analysis of the different location performed in Tab. 3 reveals that there exist significant differences between the efficiencies of banks in Slovakia and in the Czech Republic at a 1% level. A higher value of T-statistics can signify more significant differences between countries. Based on this assumption, we can say that the more significant differences were between banks in new revenue models. Based on the results of the analysis presented in Tab. 3, we can generally say that it depends on whether the bank is located in the Czech Republic or Slovakia. So we can confirm our research question that it depends on the location of the bank.

Tab. 3: Distribution hypothesis tests by location

		Traditional revenue model ρ^*	New revenue model $\overline{\rho}^*$
f(Slovakia) = g(Czech Republic)	T-statistics	12.2624	17.0163
	p value	0.0000	0.0000

Note: The functions f(·) and g(·) are (kernel) distribution functions for each model being compared.

Source: Prepared by authors

Results in Tab. 4 account for significant differences between the efficiencies of banks in different size groups at a 1% level. The significant differences can be seen in the case of traditional and new revenue model in a whole sample. Based on the results of this analysis, we can, therefore, say that it depends on whether the bank is large, medium-sized or small. So we can confirm our research question that it depends on the size of the bank.

Tab. 4: Distribution hypothesis tests by bank size

		Traditional revenue model ρ^*	New revenue model $\overline{\rho}^*$
f(Large) = g(Medium-sized)	T-statistics	8.40595	9.93722
	p value	0.0000	0.0000
f(Large) = g(Small)	T-statistics	2.28904	2.24556
	p value	0.0075	0.0075
f(Medium-sized) = g(Small)	T-statistics	5.52902	5.9401
	p value	0.0025	0.0000

Note: The functions f(·) and g(·) are (kernel) distribution functions for each model being compared.

Source: Prepared by authors

In the last part of our paper, we try to compare the differences between efficiencies calculated using two main models – traditional revenue efficiency (TRE) and new revenue efficiency (NRE) – in the whole sample, in different countries, and different sized groups. Based on the results presented in Tab. 5, we can see that there exist significant differences between efficiencies calculated by NRE and TRE model. The exception is in case of banks in small size group, where the difference is not significant, so we can claim that both models produce comparable results. In other sub-groups, it depends on whether the efficiency is calculated by using the traditional or new revenue model. As we know, a higher value of T-statistics can signify more significant differences between sub-groups. Based on this assumption, we can say that the most significant differences were between efficiencies calculated by NRE and TRE model within the banks located in Slovakia and between banks in medium-sized group. So we can confirm our research hypothesis that it depends on the applied methodology. As we can see that different method bring significantly different results, therefore it is better to use several methods simultaneously instead of a single method and try to find out models which describe the real situation in the best way.

Tab. 5: Distribution hypothesis tests by applied model

		All years
f(NRE) = g(TRE)	T-statistics	1.2449
	p value	0.0150
f(NRE, Slovakia) = g(TRE, Slovakia)	T-statistics	9.8092
	p value	0.0000
f(NRE, Czech Republic) = g(TRE, Czech Republic)	T-statistics	2.2908
	p value	0.0225
f(NRE, Large) = g(TRE, Large)	T-statistics	3.8789
	p value	0.000
f(NRE, Medium-sized) = g(TRE, Medium-sized)	T-statistics	9.4387
	p value	0.0000
f(NRE, Small) = g(TRE, Small)	T-statistics	1.94067
	p value	0.4211

Note: The functions f(·) and g(·) are (kernel) distribution functions for each model being compared.

Source: Prepared by authors

In the end, we want to apply GMM to analyse, if the bank size and location could be considered as variables with a significant impact on bank efficiency. We applied the Chow test and we found out that it was not necessary to work with panel structure; therefore, pooled OLS method was applied. Also, the standard test for OLS was applied. The heteroscedasticity was tested by studentised Breusch-Pagan test and autocorrelation by Durbin-Watson test. The results of the tests can be seen in the following table (Tab. 6).

Tab. 6: Results of GMM analysis

	TRE	NRE
Intercept	0.047545 (0.0555)	0.111233 (0.0446)**
TRE_{t-1}	0.874482 (0.0692)***	
NRE_{t-1}		0.653985 (0.0682)***
DUMMY location	0.016789 (0.0266)	0.090270 (0.0249)***
DUMMY size	0.001691 (0.0178)	-0.004375 (0.0151)
Number of observations	126	126
Multiple R-squared	0.6189	0.6563
Adjusted R-squared	0.6096	0.6478
F-statistic (p-value)	66.06 (0.0000)	77.64 (0.0000)
Breusch-Pagan test (p-value)	1.5877 (0.6622)	21.318 (0.0000)
Durbin-Watson test (p-value)	1.9469 (0.3619)	1.7596 (0.0778)

Note: Indication of significance levels: (*) 0.1 (**) 0.05 (***) 0.01. Standard error in parentheses in the independent variables.

Source: Prepared by authors

The results of the analysis pointed to the fact that in both models, the level of efficiency depends on the level of efficiency in the previous year. We could see that in both cases, the time-shifted variables were statistically significant and positive. It means that the increase in efficiency can be expected in banks, where the level of efficiencies also increased in the previous year. The dummy variables location can be considered as statistically significant only in the case of a new revenue model. We can see a statistically significant positive impact, which signalises, that it depends if the bank is located in Slovakia or in the Czech Republic. According to the coefficient sign we could expect, that with a higher value of dummy variable the efficiency increase. In our model, the higher value of the dummy variable is connected with banks in the Czech Republic. Therefore, we can suppose that Czech banks were able to obtain a higher value of new revenue efficiency. This fact could be connected to the situation on the Czech banking market compared to Slovak ones. In the case of the Czech Republic, the decline in interest rates was not so significant as in the case of Slovakia. It was influenced by the decision of the Czech national bank, who decide did not apply negative interest rates like European central bank which influence the monetary policy in case of Slovakia. Therefore, we can suppose that the decrease in interest income in the case of Czech banks were not so dramatic like in the case of Slovak banks. The last dummy variable size was not statistically significant in both models. Therefore, we can suppose that from the point of view of the regression model, it is not significant if the bank is located within the small, medium-sized or large size group.

5 Conclusion

Using the nonparametric DEA method, this paper empirically estimates the efficiencies of banks in Czech Republic and Slovakia during the period from 2008 to 2017. The original contribution of the paper is an illustrative application of the traditional Farrell (1957) DEA approach as well as a new Tone (2002) approach for evaluating the revenue efficiency of the commercial banks. From the gained results it comes out that, in the case of the traditional approach, which assumes that prices of outputs are exogenously given and also in the case when prices of outputs are added, the transformation of deposits into loans and other earning assets was successfully achieved by the larger ones on the market, and also by bank located in Czech Republic. The average traditional revenue efficiency ranged from 56.64%

to 82.68%, and average new revenue efficiency ranged from 63.11% to 82.41%.

When we look at the results of the traditional revenue model and a new revenue model, we can see a decline in efficiencies during the analysed period. As the reason of decline in revenue efficiency can be considered the decline in interest rates set up by central banks in both countries. This decline was pass-through into the interest rates on the interbank market, and also into the interest rates connected on bank products. Due to the decline in interest rates for loans, banks were not able to generate such a high value of interest income as at the beginning of the analysed period. As we know, the commercial banks try to replace this loss of interest income by decreasing operating expenses or by increasing non-interest income to prevent a decline in their profitability. We could see that the banks were able to increase their non-interest income, but the increase was not so high compared to the decline in interest income. Therefore, we can suppose that this fact had a negative impact on banks revenue efficiencies during the last years of the analyzed period.

In the next part of our paper, we examined whether there exist significant differences in estimated efficiencies. Specially, we focused on four sources of heterogeneity, namely, the type of efficiency considered, the location of the bank and the size of the bank. Based on the results of distribution hypothesis tests, we could confirm our research questions that depended on size, location and applied methodology. As the results of our analysis pointed to the existence of significant differences between banks, we also apply the GMM analysis to determine if the efficiency in the previous year and dummy variables in the form of size and location had a positive or negative impact on the revenue efficiency. The results pointed to the fact that the efficiency in the previous year has a significant and positive impact on efficiency under both models. The statistically significant positive impact in the case of dummy variables was confirmed only in the case of the location under the new revenue model. This fact could be connected to the situation on the Czech banking market compared to Slovak ones, where the decline in interest rates was not so significant like in the case of Slovakia. Therefore, we can suppose that the decrease in interest income in the case of Czech banks were not so dramatic like in the case of Slovak banks. The coefficient was significant only under the new revenue model which also take into account the information about the prices and therefore can better describe the situation on the market.

We are aware that our research has some limitations, therefore in future research, we want to include not only parameters like bank size, and location, but also extend our analysis with another bank variable as well as macroeconomic and regulatory variables.

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