DEVELOPING STUDENTS' ABSTRACT THINKING IN SECONDARY SCHOOLS

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Abstract: Abstract thinking is realized in the level of concepts, which are presented in a verbal form and are the result of a higher form of generalization and abstraction. Abstract thinking and the associated abstraction process has great importance for students' progress. Mathematics as an abstract science abstracts from the specific nature of the object, thus paving the way for new theories that are applicable in various specific practical applications. Developed abstract thinking is an important ability for students to solve problems not only in mathematics. Parameter tasks is a suitable tool for students to learn about the process of abstraction in mathematical cognition.

Keywords: abstract thinking, teaching, student, parameter, teacher

1 Introduction

Thinking generally refers to brain activity and working with data to formulate concepts, solve serious problems and questions, make inferences, and form one's own decisions. Thinking is generally considered to be the highest product of evolution and is the essence of human existence. Stríženec (2008) defines thinking as a connection between memory performance and logical abstract processing of symbols. Thinking as a mental process is a way for the exercise of human intelligence. When we think, we simultaneously perceive all the stimuli around us, classify their characteristics, and combine the relevant data (Výrost, Ruisel, 2000).

According to cognitive psychology, thinking has three distinctive characteristics:

1. Thinking takes place in consciousness, but can be inferred from observable behaviour.
2. Thinking is a process that manipulates knowledge in an individual's cognitive system.
3. Thinking is directed toward solving problems that preoccupy the individual (Mayer, 1981).

Ruisel (2008) defines thinking as a cognitive process based on the manipulation of concepts through which an individual analyzes stimuli, solves problems, makes inferences, achieves goals, and interacts with the environment.

Human thinking evolves gradually as part of human development. According to Piaget (1958), the development of thinking is divided into four stages:

1. Sensomotor stage (from birth to 2 years) - The child gradually begins to create symbols for objects in his environment.
2. Preoperational stage (up to about 6 years) - The child gradually develops abstraction. Symbols are formed not only for the external but also for the internal world. There is a significant development of language, which is of great importance for the development of symbolic activities. Words begin to represent things and concrete activity can replace thinking.
3. The stage of concrete operations (up to 10 years) - The child develops working with symbols and the ability to mentally handle objects. However, he/she can still only act on concrete objects in the immediate environment with his/her newly acquired abilities. He does not yet master more complex symbolic though operations.
4. Formal operations stage (from about 10 years of age) - The child begins to apply concrete operations to hypothetical situations. It is no longer necessary to have real objects or even their names. By moving from concrete operations to abstract ones, the young person gradually develops the skills necessary for logical reasoning. Child at this age is even able to formulate hypotheses by which he tries to explain an unknown phenomenon. Professor Hejny (1990) says that abstract strokes occur at this stage, since it is the stage where the transformation of quantity changes into quality, represented by a new knowledge or concept. The abstraction of the new concept induces a rebuilding of the knowledge structure. In fact, every discovery of a regularity or some new idea generating into a new concept is an example of an abstract stroke (Hejný, 1978).

According to Piaget's theory of human cognitive development, students around the age of 12 are already capable of abstract thinking. Abstract thinking is realised in the plane of concepts, which are presented in verbal form and are the result of a higher form of generalisation and abstraction. Abstract thinking and the abstraction process associated with has a great importance for students ' progress (not only in mathematics). Several research studies have found that students use abstraction processes to gradually acquire conceptual knowledge from previously learned practices *(*McBride, 2015; Gonda & Emanovský, 2017*)*. This finding is very important as it points to the fact that it is possible to supplement students' predominantly procedural knowledge with conceptual knowledge. This achieves the necessary balance in pupil's knowledge and to enable them to solve unfamiliar tasks independently. Professor Hejný (1990) considers the development of students' abstract thinking in conjunction with the ability to deduce to be one of the fundamental aims of mathematics teaching. At the same time, he points out the danger of students' formal cognition of mathematics being an obstacle to the development of abstract thinking. Developed abstract thinking is a prerequisite for achieving mathematical literacy, the level which is measured in OECD countries by the PISA test. Mathematical literacy is a person's ability to express, apply and interpret mathematics in a variety of contexts. It involves mathematical thinking, using mathematical concepts, procedures, facts and tools to describe, explain or predict phenomena. It helps to realize the role of mathematics in the real world, and to make sound judgments and decisions on this basis, as it is required from a constructive, engaged, and reflective citizen (Niss, 2015). In terms of problem solving, the student is expected to know basic mathematical concepts, knowledge and skills. Basic mathematical skills are considered to be: communication, visualisation/representation, strategy design, mathematisation, reasoning and argumentation, use of symbolic, formal and technical language and operations, and use of mathematical tools (PISA, 2012). In terms of problem solving, student is expected to know basic mathematical concepts, knowledge and skills. Basic mathematical skills are considered to be: communication, visualisation/representation, strategy design, mathematization, reasoning and argumentation, use of symbolic, formal and technical language and operations, use of mathematical tools (PISA, 2012). Since 2003, a persistent low level of mathematical literacy can be observed overall in OECD countries. In 2003, 66.5% of tested students in the OECD achieved a level of mathematical literacy at level 3 or below (the highest level is level 6). In 2015, 68.7% of OECD students tested reached level 3 and below in mathematical literacy. Only 4.0% of OECD students tested reached Level 6 in 2003, and in 2015 this was almost halved to 2.3%. On the other hand, Level 1 and below was achieved by 21.4% of students in 2003. In 2015, this was 23.4% of students tested. In addition to the persistently low levels of mathematical literacy, the data above also show a slight decline in the percentage of students who achieved level 6 and an increase in students at level 1 and below.

It is quite alarming that the majority of students tested do not exceed level 3 mathematical literacy. According to the results of the above mentioned testing, it seems that the main goal of mathematical education has not yet been met, which according to the SPO: "The main goal of mathematical education is for the student to acquire the ability to use mathematics and mathematical thinking in his/her future life"(SPO, 2015). If students are to use mathematical thinking in everyday life, we consider that it is necessary to set up mathematics teaching in such way that it will purposefully develop each level of thinking. From about the age of 12, it is appropriate to direct mathematical teaching primarily towards the development of abstract thinking. The acquisition of abstract thinking is the end of a developmental journey, where at the end of which stands a young person who can think logically and interact with the world around him. Already the adolescent acquires the tools that will enable him or her to move forward in the process of cognitive maturation (Ruisel, 2008).

Within mathematical symbolic notations, students are first confronted only with numbers, where prime numbers playing an essential role in mathematics (Ďuriš et al, 2021). In addition to numbers and symbols of mathematical operations, they also encounter letters later on. These letters mostly represent some numerical values. If a letter in mathematical notation represents one particular numerical value, it is called a constant. A constant denotes a fixed number whose notation is too "complicated", a number whose exact value we do not know (e.g., Ludolph's number π), or a number whose value we do not yet know. In expressions, we often encounter a letter that we call a variable. A variable represents an arbitrary number (object) from a pre-specified set. When an expression with a variable occurs in an equation, we are talking about an unknown whose specific value is to be determined so that the equation becomes an equality. Thus, we are trying to transform the equation into the form unknown = known number. Another possibility represented by a letter in a mathematical problem is a parameter. A parameter is an indeterminate but fixed element that determines the value of a variable. The paradoxical epistemic nature of this algebraic object rests on its apparent contradiction: it is a fixed concrete number, yet it remains indeterminate in that it is not a real number (Ely & Adams, 2012). To make matters worse for students, in some cases we refer to letters as coefficients (e.g., the coefficients of a quadratic equation). However, coefficients are essentially parameters.

2 Learning the concept of a parameter

The biggest problem for students is understanding the very nature and function of a parameter and the related problems of distinguishing it from the unknown in an equation. The causes of pupils' misunderstanding of the concept of parameter have been addressed by number of researchers (Bardini, Radford, & Sabena, 2005; Martinez et al, 2011; Bardini & Pierce, 2015; Emanovský & Gonda, 2020). These researches show that students often confuse the concepts of variable, unknown and parameter. A variable is often understood by students as a 'potentially determined' number. Thus, they see it as a temporarily unknown number that will be determined at some point in time. This is probably the source of the frequent confusion between the term variable and the term unknown, by which we mean an unknown number that is determined when solving an equation or inequality. Bardini et al. (2005) introduce a parameter as a new algebraic object - an unknown, but at a given time the fixed element chosen from a set of variable values. In this context, a parameter is closer to the notion of variable than to the notion of unknown. Students encounter the concept of parameter most often in secondary school within the unit of solving equations and inequalities with a parameter. A problem containing a parameter is essentially a set of problems of the same type. A concrete problem is obtained by replacing the parameter by some number. If we add a parameter to the problem, the type of the problem does not change (the quadratic equation remains quadratic). Therefore, the procedure for solving the problem is essentially the same as for a problem of the given type without a parameter. This is until the next step of the solution depends on the value of the parameter. Although the parameter in the problem statement does not make it a new type of problem requiring also a new method of solution. However, in pedagogical practice, serious problems are often encountered by pupils when solving problems with a parameter.

A new unit often evokes for students the need to learn new procedures, which is related to the prevailing tendency of students to learn mathematics by memorizing computational algorithms. The research finding is that mathematical teaching is dominated by the teaching of procedural knowledge. This is surprising because there is a largely held belief among practitioners that conceptual knowledge should be developed before students begin to acquire the relevant computational procedures and algorithms (Baroody, 2003; Kilpatrick et al., 2001). Therefore, it is important to recognise the difficulties that students have in acquiring the concept of a parameter. It is compounded by students trying to know the solution procedure without understanding the individual steps of the algorithm. Understanding the nature and function of a parameter represents a long-term task from a methodological point of view. According to Hejny (1990), it is not possible to explain to students what a parameter is in one go if they have no experience with it. This fact requires a different approach to the acquisition of the concept of a parameter by first providing students with procedural knowledge of a parameter and gradually trying to develop their conceptual knowledge of the concept of a parameter. According to several researches, this "reverse" approach of acquiring a new concept is possible (Karmiloff-Smith 1992; Siegler and Stern, 1998; Canobi 2009; McNeil et al., 2014). In acquiring the concept of parameter, student apparently has to go through this way so that subsequently the concept of parameter can be used in the development of the learners' abstract thinking.

Example 1 Solve the inequality $(x+2)(4-x) \leq 0$ on the set R.

Solution. For example, we use the zero-point method to find that the solution of the given inequality is $x \in (-\infty;-2\rangle \cup \langle 4;\infty)$. We perform the test by replacing the unknown x in the input inequality by the expression $-2-a$, where $a \in \langle 0;\infty)$. With the formed expression and the chosen admissible values of the parameter a, we verify in one computation that all elements of the interval $(-\infty;-2\rangle$ are solutions of the given inequality. To verify the correctness of the interval $\langle 4;\infty)$ we replace the unknown x in the inequality by the expression $4+a$,where $a \in \langle 0;\infty)$. These assumptions follow from the execution of the test in the previous two examples. In the first case, after indentation, we get

$$LS = (-2-a+2)(4+2+a)$$

and after modifications

$$LS = (-a)(6+a)\,.$$

The right side is equal to 0. According to the equation, LS≤ RS, it should be valid

$$(-a)(6+a) \leq 0.$$

We are looking for values of the parameter a for which the expression LS will have negative values or will be equal to zero. Again, using the zero-point method, we determine that the given condition is satisfied for the values of the parameter $a \in (-\infty;-6\rangle \cup \langle 0;\infty)$.

According to the previous examples, we expect to solve $a \in \langle 0;\infty)$. But by performing the test, we found that the condition LS ≤ RS is still satisfied also for parameter values belonging to the interval $(-\infty;-6\rangle$. It is a natural question to ask what values the unknown x , which has been replaced by the expression $-2-a$, takes for values of the parameter $a \in (-\infty;-6\rangle$. The equality of $x=-2-a$ implies $a=-x-2$ while $a \leq -6$. stands. Substituting in the inequality for the parameter a, we get

$$-x-2 \leq -6 \qquad \Rightarrow \qquad x \geq 4.$$

This confirms that the given inequality is also satisfied for values of the unknown from the interval $\langle 4;\infty)$.

By evaluating the previous performance test of the solution of the inequality using the parameter, we come to a certain difference compared to the current test of the correctness of the equations.If we have multiple solutions of an equation, the correctness test is performed by a separate calculation for each solution. If the solution of an inequality is the union of two intervals, a single calculation for both parts of the solution of the inequality is sufficient to perform the correctness test using the parameter.

We verify the universality of this "discovery" on different types on results of inequality.

Example 2 Solve the inequality on the set R $(2-x)(x^2-9) \geq 0$.

Solution. Using the zero-point method, we find that the solution of the given inequality is $x \in (-\infty;-3\rangle \cup \langle 2;3\rangle$. To perform the correctness test, we use the "discovery" from the previous Example 3.3. We perform the test by replacing the unknown x in this case by the expression $-3-a$,, where a is a parameter and represents all numbers in the interval $\langle 0;\infty)$. We will see if this also verifies the correctness of the interval, which is bounded on both sides. After fitting to the assignment and making adjustments, the following stands.

$$LS = a(a+5)(a+6)\ .$$

At the same time, according to the assignment, $LS \geq RS$ should be valid, so we solve the inequality

$$a(a+5)(a+6) \geq 0\ .$$

Using the zero-point method, we find that LS takes nonnegative values for parameter values $a \in \langle -6;-5\rangle \cup \langle 0;\infty)$. We see that the admissible values of the parameter are values from the "expected" interval $\langle 0;\infty)$, thus confirming the validity of the $x \in (-\infty;-3\rangle.\ \rangle$ part of the result. We now examine what values the variable x takes for the remaining calculated values of the parameter a. For a, the following holds

$$a = -x-3 \quad \wedge \quad (a \in \langle -6;-5\rangle \ \Rightarrow -6 \leq a \leq -5).$$

On the basis of the above

$$-6 \leq -x-3 \leq -5$$

and after modifications

$$2 \leq x \leq 3.$$

That is $x \in \langle 2;3\rangle$, which also verifies the correctness of the second part of the solution set of the given inequality.

In the following example, we examine how the test using the parameter turns out if the solution of the inequality is an interval except for one number.

Example 3 Solve the inequality $x^2(x-3) < 0$ on the set R.

Solution. The solution of the given inequality is found by means of the zero point method. All $x \in (-\infty;3)-\{0\}$ satisfy the given inequality. We perform the correctness test by exploiting the parameter by replacing the unknown x by the expression $-3-a$,, where a is a parameter that represents numbers from the interval$(0;\infty)$. The parameter is set as if the solution of the inequality were the interval $(-\infty;3)$. The goal of the following validation steps is not primarily to perform the test itself. The focus will be whether, even in this case, a single computation is sufficient to detect an inadmissible value for the unknown x in the interval $(-\infty;3)$.After replacing the unknown by the above expression with the parameter and after adjustments, the following stands

$$LS = a(3-a)^2.$$

According to the assignment, $LS < RS$ must be valid, so we solve the inequality

$$a(3-a)^2 < 0.$$

LS takes negative values for $a \in (0;\infty)-\{3\}$. In a simple way, we find that if $a=3$, then $x=0$. It can be stated that even in this case, one calculation is sufficient to verify the correctness of the result of the inequality as a whole.

In the final example, we will verify whether one calculation is sufficient to test correctness even if the inequality has a solution consisting of more than two parts.

Example 3 Solve the inequality on the set R $\frac{(x-3)(x+4)}{x(6-x)} \leq 0$.

Solution. Using the zero-point method, we arrive at a solution of the inequality $x \in (-\infty;-4\rangle \cup (0;3\rangle \cup (6;\infty)$, which can be naturally split into three subsets. We construct an expression to replace the unknown as if we were going to check the correctness of only one of the intervals, for example the interval $(6;\infty)$. All elements of this interval can be replaced by the expression $6+a$ where a is a parameter that can be replaced by any number from the interval $(0;\infty)$. After the above substitution and after adjustments, the following stands

$$LS = \frac{(a+3)(a+10)}{-a(a+6)}\ .$$

From the definition of the inequality $LS \leq RS$, so we solve the inequality

$$\frac{(a+3)(a+10)}{-a(a+6)} \leq 0\ .$$

LS takes non-positive values for $a \in (-\infty;-10\rangle \cup (-6;-3\rangle \cup (0;\infty)$. We know that for $a \in (0;\infty)$ the unknown takes values from the interval $(6;\infty)$. Based on the experience from the previous examples, we assume that to complete the correctness test, it is sufficient to find out what values the unknown takes if the parameter represents numbers from the intervals $(-\infty;-10\rangle$ a $(-6;-3\rangle$. Based on the substitution used, $a = x-6$. The interval $(-\infty;-10\rangle$ can be replaced by the inequality $a \leq -10$, after substituting for the parameter and after adjustments we get $x \leq -4$,, which corresponds to the first part of the solution, i.e. the interval $(-\infty;-4\rangle$. The interval $(-6;-3\rangle$ can be replaced by the system of inequalities $-6 < a \leq -3$. Substituting for the parameter and after simple modifications, we have the notation $0 < x \leq 3$.. This notation corresponds to the second part of the solution of the inequality, which is the interval $(0;3\rangle$. This confirms to us that even for a result of an inequality that consists of more than two parts, it is sufficient to set the parameter to cover one part of the result in order to perform the overall test.

3 Process of abstraction

The aim of the previous examples is to learn the notion of parameter and to get the first experience for students with this mathematical concept. It is a suitable tool for the process of abstraction. The correctness test is only a "mathematical backdrop" to introduce the new concept into the students' world of knowledge. As we mentioned above abstract thinking is realized in concepts that are presented in verbal form and that are the result of a higher form of generalization and abstraction. Abstract thinking and the abstraction process associated with it, has a great importance for students' progress. Mathematics as an abstract science abstracts from the concrete nature of an object, thus paving the way for new theories that are applicable in a variety of concrete practical applications. Abstractness allows mathematics to capture the variety of forms of real objects and consequently to reveal the relatedness between different objects (Šedivý et al., 2001). The parameter problems are a good tool for students to learn about the process of abstraction in the context of mathematical cognition. With this intention, parameter tasks will become a powerful tool for developing students' abstract thinking because they will be directly initiated into the mathematical mode of abstraction. The process of abstraction can be made accessible for students in the form of graded tasks, with the gradual incorporation of a parameter or several parameters into an initially 'concrete' task. When a parameter (or multiple parameters) is used in solving a particular problem, there is a move away from calculating that particular problem to finding a solution to that problem in general. With parameters, it is possible to discover various dependencies between values affecting the final outcome of the problem. A common product of "parameterizing" a problem is a formula that can be "entered" into a computer, which will do the necessary calculations after retrieving the specific values of each parameter. Parameterization of the task leads to the creation of a solution for the entire task system. The ability to parameterize a problem and then solve the resulting parametric problem is a significant benefit and an extension of students' problem solving skills and abstract thinking. Another benefit of problems solved by parameterization is the further development of the correct concept of a parameter as a substitute for numbers, in the search for an efficient way to solve the given problem. Teaching mathematics enriched with graded problems associated with parametrization develops students' ability to solve a specific problem with general insight. Through the parametrization of problems, students become aware that a particular equation is a mathematical model of a given specific situation, but the corresponding parametric equation is a mathematical model of the whole system of problems. We think that this knowledge has a great benefit, for example, for future programmers. Their task is often to create a complex solution to a problem (preferably creating a formula), which is the core of a given program. The computer retrieves the necessary values (parameters) from the requestor of the specific request and offers the desired result in a short time. This is an example where a person has created a solution method and then "taught" the computer to calculate specific tasks from the given task system. This fulfils society's requirement for teaching of mathematics: man creates - computer computes. In the following we present some sample graded problems associated with parameterization, which aim to support the development of students' abstract thinking.

4 Conclusion

We are living in the time of the fourth industrial revolution (IR 4.0), which is changing the way we live, work and communicate at a relatively rapid pace. This trend is likely to continue. According to the World Economic Forum, an estimated 65% of children enrolling in primary education today will end up working in jobs that have not yet been created at that time. At the same time, there is constant emphasis on education being geared towards preparing young people for their future working lives. Thus, education needs to be responsive to the current needs of practice. The response to the needs of IR4.0 is the educational vision of Education 4.0, whose primary objective is to match human skills and new technical capabilities in order to prepare students for new opportunities in the constantly evolving labour market. In today's companies, humans are needed to be able to consider a lot of disparate information and combine it into a single solution. The expected creativity of a graduate is linked to divergent thinking, which opens the mind to the knowledge that there may be multiple possibilities of the right way to solve a problem. Given the relatively easy and quick availability of the necessary information, the Education 4.0 vision recommends that education should focus more on developing the skills that will enable students to actively use the information they have acquired. This is the brain's ability to switch seamlessly between different modes of thinking, e.g. creative, abstract, critical thinking, etc. The more fluently you can do this, the more likely it is that new patterns and associations will be formed. This ability requires sufficiently developed abstract thinking so that the graduate trainee (future employee) is able to abstract important knowledge from the situations and objects which he or she acquired and is able to adapt to new, often very distant, conditions. Teaching mathematics with a focus on the development of critical thinking creates the preconditions for being able to transfer the acquired knowledge from one object to another, which can be regarded as signs of cognitive flexibility. Cognitive flexibility helps to maintain attention or to shift it to something else depending on the changing demands of the environment or to take a different approach to different situations. At the same time, students are expected to solve problems independently with which they have not encountered before. To do this, they need to be able to form an overall picture of the problem in the context and, through the process of abstraction, to identify the essential features of the problem and relate them to other abstractions in their minds. An important factor is the ability to notice details, which are often crucial in finding the right solution to a problem. It is the parameterization of the problem, i.e. the attempt to solve the problem in a general way, that often leads to the discovery of these key details, which are often the decisive factor for the effective solution of the given problem. We believe that current mathematical education does not need primarily a change in content, but rather a change in the forms of teaching and the goals of mathematical education. In the upgraded ISCED 3 state curriculum for upper secondary education, the term 'parameter' no longer appears. Thus, the teaching of problems with a parameter has been omitted from the mathematical teaching. The notion of parameter is a difficult concept, but it has potential to develop the necessary abstract thinking and in our opinion, a sufficient argument for its reintroduction into teaching of mathematics.

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